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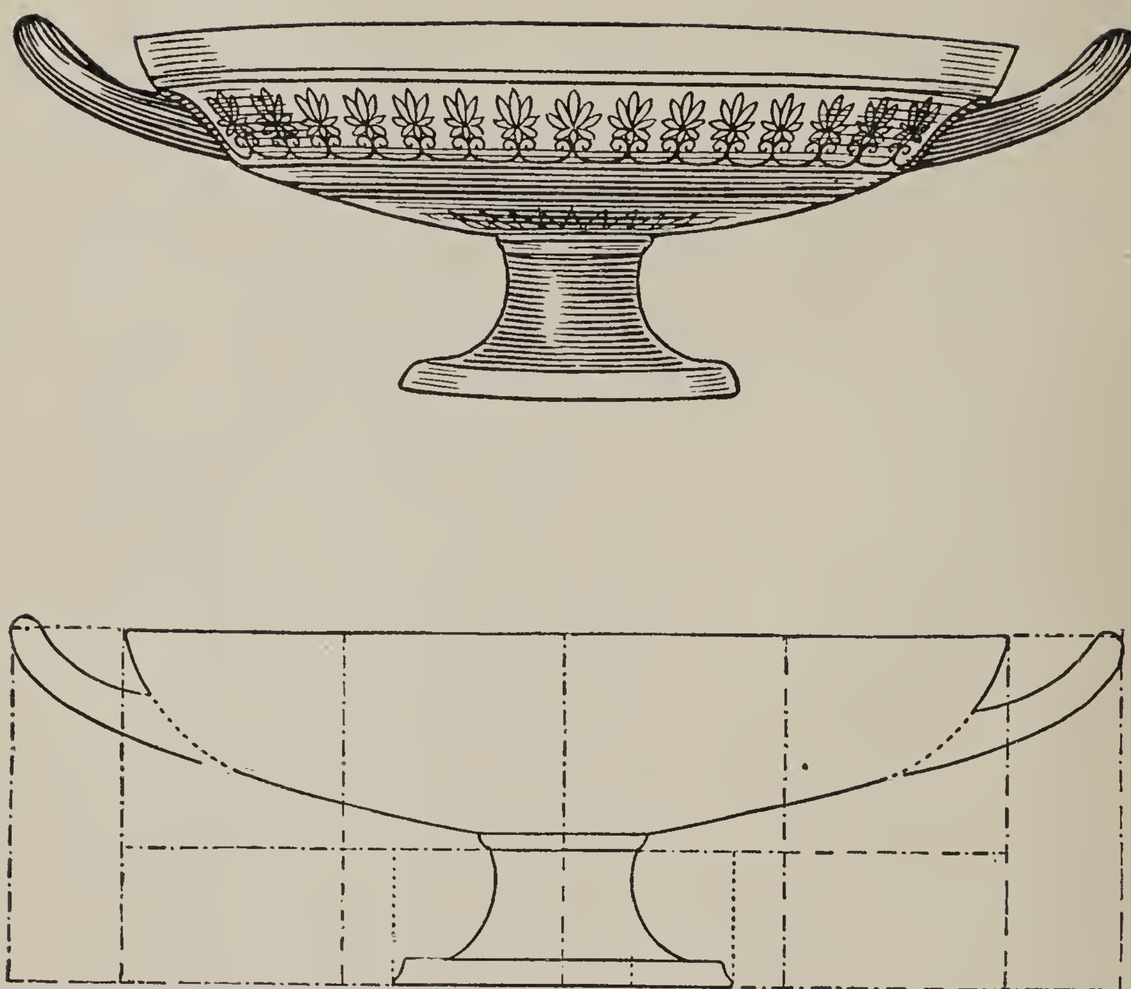

A Greek Vase and the Scale Drawing upon Which the Design Is Based

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JUNIOR MATHEMATICS

BOOK ONE

BY

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AUTHOR'S PREFACE

The study of certain phases of mathematics deserves an important place in a liberal education. Modern thought and enterprise are steadily increasing in mathematical precision, as is apparent in the statistical aspects of the biological and human-nature sciences. To understand modern civilization in many of its aspects one must be able to appreciate its precise quantitative character. One must be able to read intelligently quantitative accounts of modern enterprises. Familiarity with modern methods of measurement and skill in doing quantitative thinking are essential. This text offers a *general* course intended for those who continue studies in the senior high school as well as for those who do not continue. It aims to contribute to the pupil's liberal education by preparing him to understand the quantitative aspects of contemporary civilization.

Since this is a first course in junior high school mathematics, it presupposes knowledge of the fundamental operations with whole numbers, with common fractions, and with decimals. As soon as proficiency in these operations has been attained, the pupil is prepared to take up the study of this course.

The following aims have been set up in the selection and organization of material:

1. The material used must fill a real need in the life of the pupil. It must be useful to him in his present

school studies as well as in preparing him to perform activities in later life. It must have social value.

In this textbook the social realities of the material are emphasized throughout. Free use is made of discussions, pictures, and diagrams to make the social worth of the material appeal to the pupil.

2. The material must be adapted to the abilities of pupils of the early adolescent period and lie within their experience.

Since the success of a course must depend largely on the extent to which this adaptation is accomplished, all the material has been tried out with junior high school classes. On the basis of results of carefully made tests, subject matter not adapted to the mental ability of these pupils has been rejected or changed in treatment. Thus, business applications have been limited to such matters as pupils may be expected to appreciate and understand. Ideas of percentage, interest, etc., which everybody should know, are presented concretely. This is considered sufficient for the majority of pupils. If in a school there are enough pupils intending to enter commercial work, they may be given an additional vocational course in commercial arithmetic. It is advised that vocational courses be given near the end of the junior high school course, or in the senior high school, rather than in the first year.

3. It is not sufficient to teach mathematics merely as a body of principles. There ought also to result training in mathematical methods of thought, effective habits of study as applied to mathematical situations, a conviction of the universal applicability of powers of concentration, and insight into the method of sound

generalization in any field. Such larger values cannot be depended upon to come of themselves. Their achievement has been a matter of constant attention throughout the course.

4. Quantitative relations are to be studied in three ways: geometrically, as in length, area, and graphs; algebraically, as in formulas, equations, and functions; and arithmetically, as in tables and evaluation.

Geometry in its simplest form, because of its usefulness and concreteness, has been made the core of the course. It is experience getting in space relations. It is intuitional, experimental, constructional, not demonstrative. The principles established are those that appeal to the pupil as valuable information. This experience in geometry makes it possible to make the beginning of topics in algebra concrete and then to pass from the concrete to the abstract.

Algebra is not taught in this text as an organized science, but as a helpful tool in the study of other topics. Formulas and equations are the outcome of concrete problems and relate to real things. Since the geometry is concerned only with the conception of plane figures, such as line segments, angles, and areas, no algebraic functions of degree higher than the second are introduced. These functions are to be considered in the second and third course where three-dimensional figures are introduced and where algebra is studied as a science.

An abundance of work in arithmetic has been provided. The arithmetic aims to secure proficiency in the fundamental manipulative processes needed in the course of ordinary life and in the acquisition of further

mathematical knowledge. Arithmetic is reviewed mainly through applications found within the domain of child life, through wide experiences in many situations. Thus, the arithmetic has changed from the formal drill process used in the lower grades to a process of assimilation through application.

The following are some of the important features of this text:

The material is organized in pedagogical units, rather than in logical units, *i.e.*, material has been put together which is most economically and effectively learned together. Instead of learning a number of isolated facts or lessons, the pupil sees the relations of a compact body of facts closely related to one another and to the major topic. He will therefore not only master the unit with economy of effort, but will retain it more permanently than when facts are studied separately.

The method of approach is inductive. This is the method of the beginner. The main object is thorough understanding of the concepts to be derived from numerous instances familiar to the pupil by means of examining, contrasting, and comparing. Formal statements, defining the new term or stating the principle, are always the last step in the development.

New terms are introduced when they are needed, and not earlier. Moreover, invariably such explanations are given in the text as are necessary to let both the teacher and the pupil understand the reason for bringing in new terms. Each new topic is started with a real problem impressing the pupil with the usefulness of the subject to be studied.

The book contains a great many type examples, illustrating both the method and the arrangement of a solution. Neat and clean-cut written work in mathematics is an important factor in clean-cut thinking. Hence this feature is stressed throughout.

The language of the book is precise, but the sentences are simple so that the pupil can read and understand them. Problems are arranged in order of difficulty determined from actual pupil-performance. There is not a problem in the book which has not been worked by pupils.

In the process of finding the solutions of a problem and in the interpretation of the solutions, it is important that the pupil understand the meaning of approximated measures. Lack of appreciation of the degree of precision causes not only misleading impressions, but also useless effort and waste of time. Since measurement plays an important part in this course and since many data are necessarily approximations, the pupil must learn to determine the degree of precision in the results determined from these data. Hence, considerable attention is given to this matter.

The author wishes to express his appreciation to Director Chas. H. Judd and to Professor H. C. Morrison and the late Professor S. C. Parker of the School of Education, the University of Chicago, for work on the manuscript and for their constant advice and inspiration during the process of development of this course, and to Professor W. C. Reavis, whose interest and support as principal of the University High School has greatly facilitated experimentation in junior high school classes.

The studies which contributed to the refinement of the material used in the course presented in this book were aided by a grant from the Commonwealth Fund.

E. R. Breslich.

CONTENTS

INTRODUCTION

The junior high school is recognized by its most ardent advocates as an immature institution. It has grown up during the last decade in all parts of the United States with astonishing rapidity. It came into being because there was a general demand for more productive teaching in the later years of the elementary school and a better administration of the social and personal lives of children in their early adolescent years. It came into being before suitable materials of instruction were at hand and indeed before those who recognized the need for a new institution were altogether certain what the characteristics of this new institution should be.

There are a few instances on record where the junior high school has been tried and abandoned because those who inaugurated it did not know what to put into the courses of study or how to relate this unit to the established units of the school system. There are numerous other instances where the junior high school has been only moderately successful because it was not adequately organized and equipped to fulfill its special functions.

As experience has accumulated, it has become increasingly evident that the ultimate success of the junior high school depends in a very large measure on the preparation of a new kind of teaching material.

This material must be made up of the fundamentals of high school subjects so arranged that they will gather up and review all of the results of elementary education and at the same time open the way to adult life either in the practical world or through higher and more completely differentiated studies of the senior high school. It is not enough that high school courses be carried back into the earlier years of the school's work, nor is it a solution of the junior high school's problem to mix, without genuine intellectual coördination, some of the materials which have heretofore been taught in the seventh and eighth grades with a few of the exercises which used to be given in the ninth grade or higher in the school curriculum. There must be a new and completely integrated body of instructional material capable of bridging the gap which has up to this time separated the elementary grades from later intellectual life.

The field of mathematics was one of the first in which teachers began to experiment in the effort to work out a true combination of elementary and higher materials. Numerous books have appeared which more or less successfully accomplished the purpose of introducing pupils to the fundamentals and applications of all the mathematical sciences.

Professor Breslich has a number of advantages in entering this field of experimentation. His books on combination mathematics for the senior high schools are far and away the most successful books of that type in the English language. They have been extensively used in the ninth year and upper years of the high school. Furthermore, Professor Breslich has had the

opportunity for the last six years of working with a seventh year class in the laboratory school of the University of Chicago where, through actual experimentation with a succession of classes which he has himself taught, he has refined his methods and materials to the point where they can now be offered to a wider constituency.

Two years ago, through a subsidy from the Commonwealth Fund, Professor Breslich was enabled to visit the leading junior high schools of the country and to make a study of their courses in mathematics. He has also conducted courses in the principles of teaching mathematics in the School of Education of the University of Chicago, and has in this way come into contact with experienced teachers and supervisors from all parts of the country.

This book is accordingly one of the maturest courses for junior high schools that has been prepared. It follows lines which Professor Breslich has long advocated in articles in the *School Review* and elsewhere and lines which the National Committee on Mathematical Requirements accepted in its report of 1923. It is full of exercises suited to the interests and intellectual abilities of adolescent children. It is a true fusion of arithmetic, geometry, and algebra. It has a background of careful sifting through practical use and criticism from a large number of teachers and students of the junior high school.

This book is to be followed by another, designed for use in the later years of the junior high school. The second book is completed and is equally based on trial and criticism. This and the second volume of the

series are presented by the institution which Professor Breslich represents, with the full confidence of his associates who have watched the development of his work and shared, to some extent, in its criticism.

August 7, 1924 CHARLES H. JUDD.

JUNIOR MATHEMATICS

CHAPTER I

WHAT IS MEANT BY A LINE SEGMENT

How to Measure Segments with a Ruler

1. The importance of measurement. Mathematics is one of the earliest sciences. All people have found it necessary to measure. Even the savages had to learn how to count and to measure their food supplies and other necessities. This was the beginning of mathematics. At first people measured in a very crude way by means of fingers or pebbles, but as civilization progressed, better and more accurate methods of measurement were needed and therefore developed. Today, everybody uses some kind of measurement in his daily tasks. Without the use of mathematics, we could not construct our modern machinery, our great buildings, railroads, bridges, or ships. We could not even carry on our daily business. Since modern civilization owes so much to mathematics and depends on it so largely, every pupil should study this subject.

Measurement plays a large part in mathematics because it is something everybody needs to know about. The two boys in Fig. 1 are using measurement in laying out a tennis court.

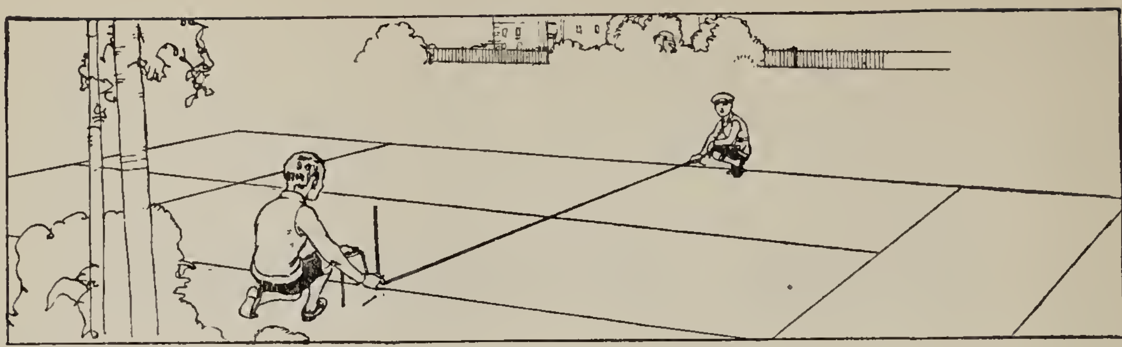


FIG. 1

A boy who plans to make a radio set, a footstool, a kite, or a canoe must determine by measurement the amount of material he needs. A girl who is going to make an apron, a doily, a tablecloth, or a dress determines by measurement the required amount of material before buying it. She knows from experience that failure to measure may cause waste of material and money if she should purchase more than she needs.

In making a cake it is safer to use a measuring cup and scale than to guess at the amount of the ingredients. When we are sick, the doctor measures our temperature with a thermometer. We use clocks to measure the time, a gas meter to tell the amount of gas we use, a speedometer to show how fast our car travels, and a steam gage to determine the pressure in our heating plant.

The accuracy required in measurement depends largely on the use to be made of the result. If a boy wants to measure the length of the block in which he lives, he may determine it by simply stepping it off; but the surveyor needs to measure the block with great care using as an instrument of measurement—a well-made steel tape. A carpenter can measure sufficiently well with a yardstick or tapeline, but the

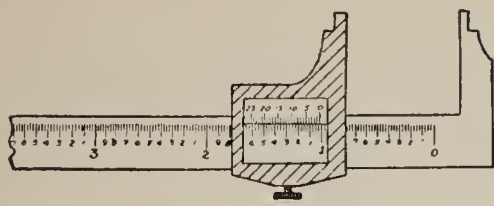


Fig. 2

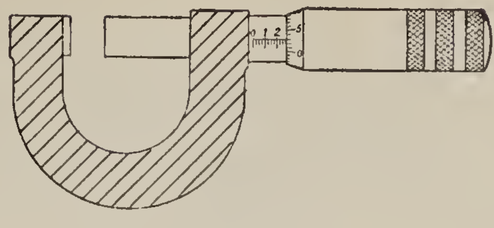


Fig. 3

machinist finds it necessary to use a vernier caliper (Fig. 2) and a micrometer (Fig. 3) to measure a required length.

Exercises

1. Make a list of the measuring instruments used in your household, and tell how each is used.

2. If you are studying science, make a list of the measuring instruments used in that course and tell what they are used for.

3. Name some measuring instruments that are used in stores, offices, and factories.

2. The meaning of a straight line. We shall begin measuring by finding lengths of straight-line distances, because lengths laid off on straight lines are very simple to measure. A very good idea of what is ordinarily called a straight line may be obtained from the following:

Fold a sheet of paper and crease it by moving a finger along the fold. The crease of the paper represents a *straight line.* Boundary lines are frequently straight lines. Thus, the edge of a good ruler, the edges of a sheet of notebook paper, and the boundaries of a window pane are examples of straight lines. When we study lines in *geometry* we do not consider width, thickness, color, or weight. Geometric lines have only length, and we are interested mainly in measuring lengths.

Exercises

1. Point out illustrations of straight lines in the classroom.

2. State some examples of straight lines found outside of the classroom.

3. Can you mention some examples of straight lines found in nature?

3. How to make drawings representing straight lines. Various instruments may be used to make drawings representing straight lines. Fig. 4 is a picture of a *ruler*. The edge AB is a straight line.

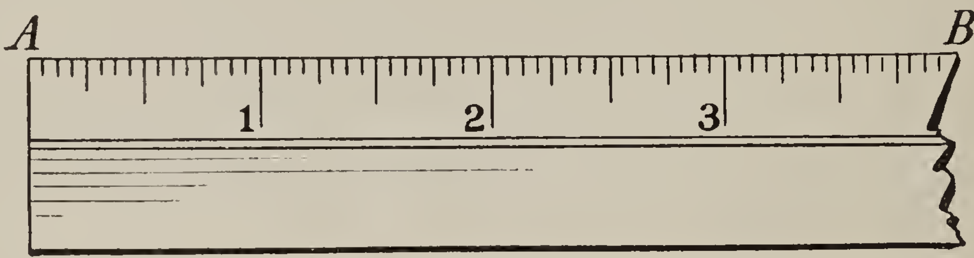


Fig. 4

If the marks or graduations are omitted from a ruler, it is called a *straightedge* (Fig. 5).

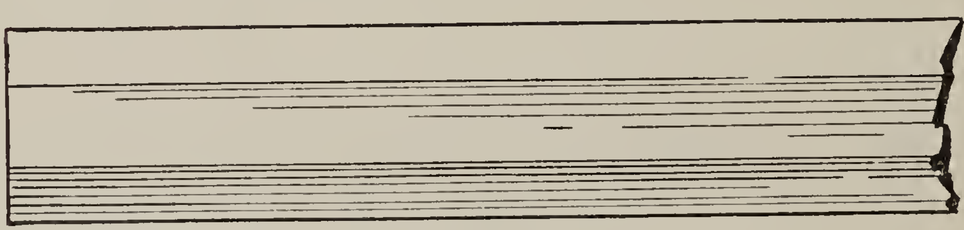


Fig. 5

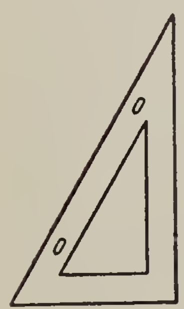


Fig. 6

Other instruments used to draw straight lines are the *triangle* (Fig. 6) and the *T-square* (Fig. 7).

We shall now learn how to use the ruler in drawing straight lines.

Fig. 7

Exercises

1. Place a ruler on a sheet of paper (Fig. 8) and move the point of a sharp pencil along the edge making a line on the paper. The drawing obtained is said to *represent* a straight line. To be brief, we shall call it a straight line, but strictly speaking, it is not a geometrical line because it has width.

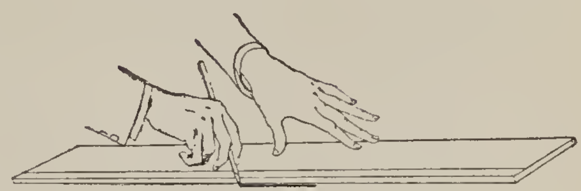


Fig. 8

2. The drawings (Fig. 9) are formed by straight lines. Study

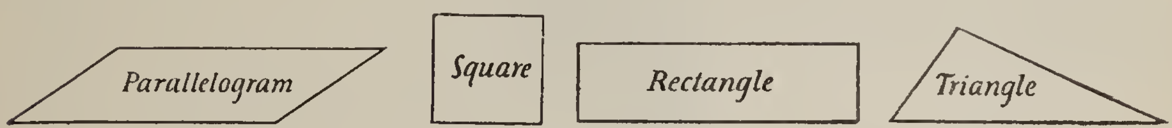


Fig. 9

the shapes and remember the names. Then, without looking at the figures, draw others like them on a sheet of notebook paper.

Find other figures of such shapes in the classroom.

3. Make drawings like those shown in Fig. 10.

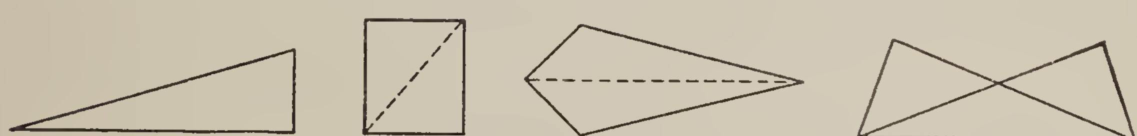


Fig. 10

4. By means of a ruler or straightedge, test whether or not a surface is *plane* (flat).

Suggestion: Place the straight edge of the ruler on the surface in a number of positions (Fig. 11). If for every possible position the edge lies completely on the surface, the latter is said to be *plane*. Apply this test to your desk; to a table top. A carpenter when making a plane surface uses this method of testing.

4. Points. Small dots made on paper with a sharp pencil, or chalk dots made on the blackboard, are ordinarily used to *represent* points. Two geometric lines cut each other in a point, such as point A (Fig. 12). Since geometric lines do not have width, it follows that *geometric points* do not have length, breadth, or thickness. They only indicate position. Points are usually denoted or named by *capital* letters.

Fig. 11

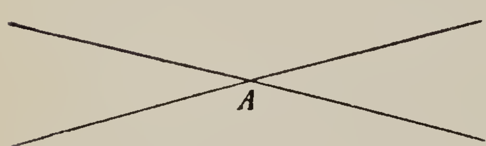


Fig. 12

Exercises

1. Draw two *intersecting* (crossing) straight lines. From the figure tell how many times two straight lines can intersect.

This exercise illustrates the fact that *two straight lines can have only one point in common.* The point in which two straight lines intersect is their *point of intersection.*

2. Draw four straight lines passing through a fixed point A. How many straight lines may be drawn through A?

This exercise illustrates the fact that *through a given point any number of straight lines may be drawn.*

3. We know from geography that lines drawn on a map from right to left are *east-west* lines. *East* is to the *right; west* is to the *left*. Make a drawing like Fig. 13, and through each of the points A, B, C, D, E, draw the east-west line.

4. In the kind of drawing or diagram which we call a map, *upward* means *north*, or toward the top of the page, *downward* means *south*, or toward the bottom of the page. Make a drawing like Fig. 14, and through each of the points A, B, C, D, E, draw the north-south line.

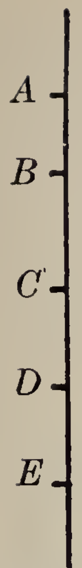


FIG. 13

FIG. 14

5. To draw straight lines it is necessary to have a good ruler. Test the straightness of your ruler as follows:

On the blackboard or on a sheet of paper mark two points, A and B.

Placing the ruler upon the blackboard or the paper so that the edge passes through A and B, draw a line through A and B.

Then place the ruler on the opposite side of the line AB, making the edge again pass through A and B, and draw a line through A and B.

If the edge of the ruler is straight and if the drawing is well made, the second line should fall exactly on the first. The two lines are then said to *coincide* (fall together).

This exercise illustrates the fact that *through two given points only one straight line can be drawn.*

5. Line segment. In the preceding paragraphs the word "line" has been used without considering length. A geometric line is unlimited in length.

FIG. 15

A *limited portion* of a line, *i.e.*, one which is *bounded* by two points like A and B (Fig. 15), is a *line segment.*

Note that the difference between line segment AB and line AB is that the first is bounded by two points, while the second extends indefinitely.

6. How to measure length. Lengths may be measured with various instruments. To measure the length of a line segment as AB (Fig. 16) with a ruler,

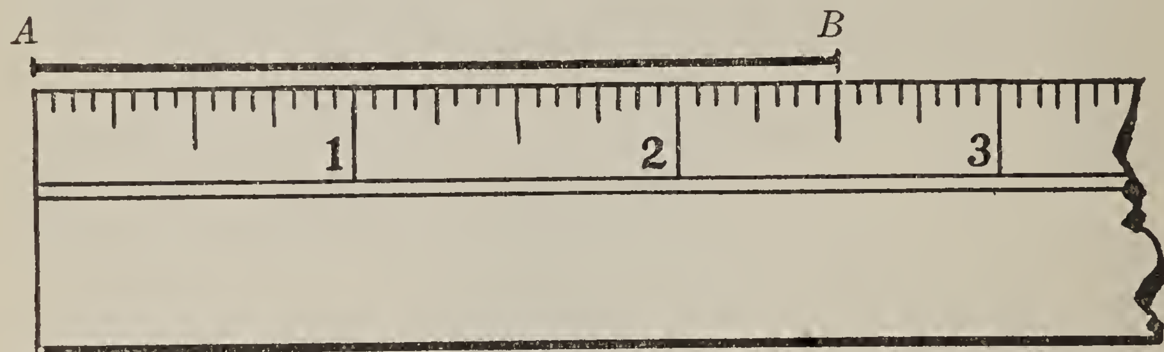


FIG. 16

place the edge, which is marked off into equal parts, along AB with the zero mark directly under A.

Then read off the mark on the ruler which is directly under B. If the ruler is graduated in inches the reading gives the number of inches contained in AB. This number is the *length* of AB, and the inch is the *unit segment*. Thus, to *measure* a line segment is to determine how many times it contains another segment, called the *unit segment*.

EXERCISES

1. Look again at Fig. 16 and tell the length of AB.

2. Draw a line segment and find the length by measuring with a ruler as you were shown in §6.

In this exercise some of you have discovered that when we are measuring segments, we cannot always find the *exact* length, because the end point of the segment does not in every case fall

exactly over a mark of the ruler. The length then has to be *estimated.* Thus, in Fig. 17 the length of AB is greater than $2\frac{7}{16}$ and less than $2\frac{1}{2}$. It seems to be nearest to $2\frac{7}{16}$. The length is said to be $2\frac{7}{16}$ *approximately*, or $2\frac{7}{16}$ *to the nearest sixteenth of an inch.*

Briefly, $AB = 2\frac{7}{16}$ approximately.

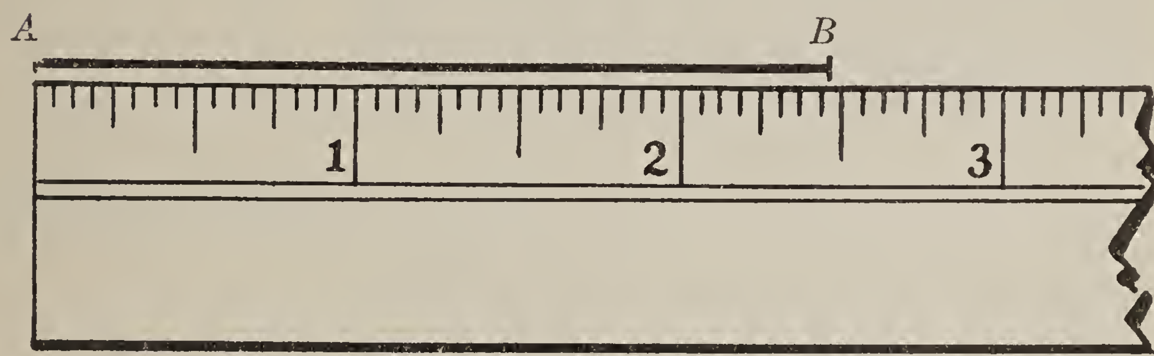


Fig. 17

3. State how one measures a line segment with a ruler.

4. Draw a line segment and find the length to the nearest sixteenth of an inch. Write the result in the form used in the final statement in Exercise 2.

5. Measure each of the segments AB, BC, CA (Fig. 18). Write the results arranged as follows:

$AB =$

$BC =$

$CA =$

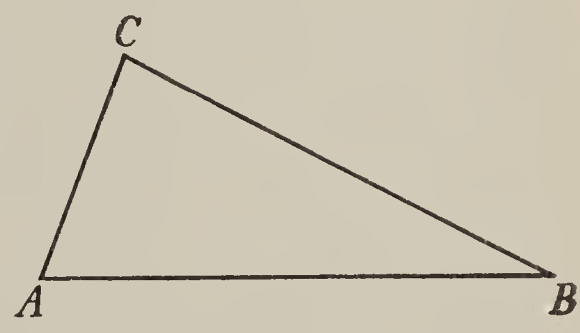


Fig. 18

Add the lengths of the three segments and divide the sum by 3.

6. Make a drawing like Fig. 19. Measure each of the three segments. Find the sum and divide it by 3. Arrange the work as in Exercise 5.

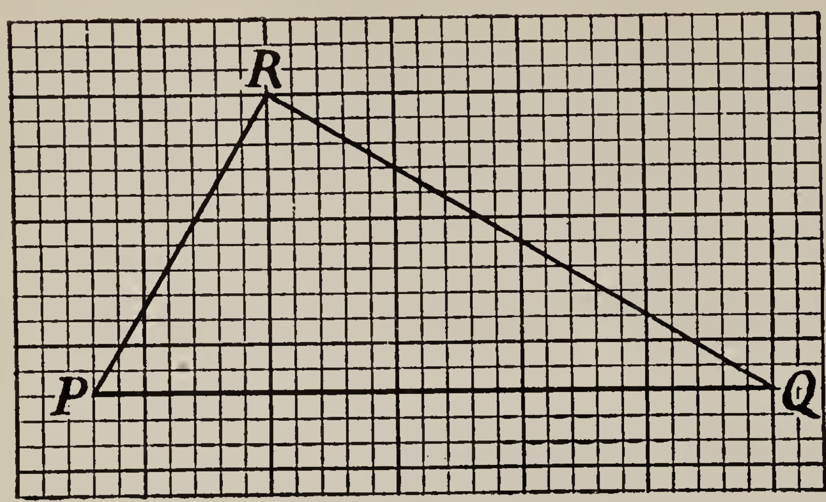


FIG. 19

7. Draw five segments and name them AB, CD, EF, GH, IK. Measure each to the nearest sixteenth of an inch. Find the sum and divide the sum by 5.

8. Make a drawing like the rectangle in Fig. 9 and measure each side to the nearest sixteenth of an inch.

9. Draw a square (Fig. 9) and measure each side to the nearest sixteenth of an inch.

7. Arithmetical average. In a problem in measuring, the results of all pupils in a class usually do not agree exactly. Some pupils work more accurately than others. If we compare classes, we find that some classes work more accurately than others. If we add the results when all pupils in a class have measured a line segment, and then divide the sum by the number of pupils in a class, we obtain the *average length,* or the arithmetical average. Likewise, the arithmetical average of *several* segments is found by adding the lengths of all the segments and dividing the sum by the number of segments. The average of the results obtained by all the pupils in a problem in measurement is used to tell how accurately the class can measure. We may also find the separate averages for the boys and the girls and, by comparing each of these averages with the correct measure, we can compare the work of the boys with that of the girls.

The following example shows how to find the average of several numbers.

Find the average of $6\frac{2}{3}$, $4\frac{1}{2}$, $5\frac{3}{4}$.

Solution: Changing all fractions to the same denominator, we have

$$6\tfrac{2}{3}+4\tfrac{1}{2}+5\tfrac{3}{4}=6\tfrac{8}{12}+4\tfrac{6}{12}+5\tfrac{9}{12}$$
$$=16\tfrac{11}{12}$$

To find the average, divide $16\frac{11}{12}$ by 3.

$$\text{Average}=\tfrac{1}{3}\times 16\tfrac{11}{12}=\tfrac{1}{3}\times\tfrac{203}{12}$$
$$=\tfrac{203}{36}$$
$$=5\tfrac{23}{36}$$
$$\text{Average}\quad=5\tfrac{23}{36}$$

Computation:

$$\begin{array}{r} 16 \\ \underline{12} \\ 32 \\ \underline{16} \\ 192 \\ \underline{11} \\ 203 \end{array}$$

$$\begin{array}{r} 5 \\ 36\overline{)203} \\ \underline{180} \\ 23 \end{array}$$

Before adding the fractions in the preceding example we had to change them to fractions having a common denominator. The following is a simple way of finding the *least* common denominator of several fractions, as $\frac{1}{5}$, $\frac{3}{10}$, and $\frac{1}{8}$.

1. Choose the largest of the denominators, *i.e.*, 10.

2. Multiply it by 2, then by 3, etc., until a multiple of 10 is found which contains each of the denominators as a divisor. Thus, 2×10 and 3×10 contain 5 but not 8. However, 4×10 contains both 5 and 8. It is the required least common multiple of the denominators.

Similarly, to find the least common denominator of $\frac{3}{16}$, $\frac{17}{48}$, and $\frac{8}{9}$ we write down the largest denominator,

48. Since 48 contains 16, we disregard the 16. Multiplying 48 by 2, 3, etc., we see that 3×48, or 144, contains 9. Thus 144 is the least common denominator.

Exercises

For each of the columns below find the average. Arrange the work as shown in the preceding example.

1.	2.	3.	4.	5.
$8\frac{1}{2}$	$32\frac{7}{8}$	$16\frac{2}{3}$	$8\frac{1}{2}$	$22\frac{3}{16}$
$10\frac{1}{4}$	$20\frac{3}{4}$	$15\frac{1}{3}$	$15\frac{3}{4}$	$17\frac{29}{48}$
$15\frac{3}{8}$	$12\frac{21}{24}$	$19\frac{5}{6}$	$22\frac{3}{8}$	$10\frac{8}{9}$
	$4\frac{5}{6}$		$7\frac{2}{3}$	

6. In measuring a line segment six pupils of a class found the following results:

$4\frac{1}{2}$, $4\frac{7}{16}$, $4\frac{9}{16}$, $4\frac{3}{8}$, $4\frac{7}{16}$, $4\frac{1}{2}$. Find the average.

7. Draw five segments, each being longer than the preceding. Name them AB, CD, EF, GH, and IK. Measure each segment to the nearest sixteenth of an inch. Subtract their lengths in the following order: the first from the second, the second from the third, the third from the fourth, and the fourth from the fifth. State your results in the following form:

$$CD - AB = \quad, \qquad GH - EF = \quad,$$
$$EF - CD = \quad, \qquad IK - GH = \quad.$$

Exercises 8 to 28 give further practice in adding and subtracting fractions.

8. In the following subtract the lower number from the upper. Check each by adding the lower to the difference.

$$\begin{array}{r} 12\frac{1}{4} \\ 7\frac{2}{3} \\ \hline \end{array}$$

Solution:

$$\begin{array}{rcl} 12\frac{1}{4} = 12\frac{3}{12} = 11\frac{15}{12}. \\ 7\frac{2}{3} = 7\frac{8}{12} = 7\frac{8}{12}. \\ \hline \text{Difference} = 4\frac{7}{12}. \end{array}$$

Check:

$$7\frac{8}{12} + 4\frac{7}{12} = 11\frac{15}{12}$$
$$= 12\frac{1}{4}$$

9. In each of the following subtract the lower from the upper number·

$12\frac{3}{16}$	$18\frac{7}{12}$	$50\frac{1}{3}$	$7\frac{1}{2}$	$68\frac{3}{4}$
8	$10\frac{1}{3}$	$14\frac{11}{12}$	$5\frac{3}{4}$	$11\frac{45}{48}$
$9\frac{7}{8}$	14	$8\frac{2}{7}$	$22\frac{1}{3}$	$14\frac{1}{2}$
$5\frac{2}{3}$	$8\frac{2}{3}$	$4\frac{4}{5}$	$9\frac{3}{5}$	$7\frac{1}{3}$

Find the following sums and differences. Arrange all work as shown in Exercise 10.

10. $\frac{1}{2}+\frac{3}{4}-\frac{1}{5}$

Solution: $\frac{1}{2}+\frac{3}{4}-\frac{1}{5}=\frac{10}{20}+\frac{15}{20}-\frac{4}{20}=\frac{21}{20}=1\frac{1}{20}$.

11. $\frac{1}{3}+\frac{1}{4}$

12. $\frac{1}{4}-\frac{1}{6}$

13. $\frac{5}{6}+\frac{7}{9}$

14. $\frac{7}{12}-\frac{1}{3}$

15. $\frac{7}{8}-\frac{1}{4}$

16. $\frac{7}{12}-\frac{1}{5}$

17. $\frac{9}{10}-\frac{2}{5}$

18. $1\frac{2}{3}+3\frac{1}{2}$

19. $2\frac{1}{2}+1\frac{3}{8}$

20. $8\frac{1}{8}-6\frac{1}{4}$

21. $2\frac{1}{2}+2\frac{2}{3}-1\frac{1}{8}$

22. $4\frac{4}{7}+6\frac{7}{8}-2\frac{1}{2}$

23. $10\frac{1}{2}-8\frac{1}{3}+5\frac{3}{4}$

24. $18\frac{3}{16}-10\frac{4}{5}+\frac{1}{2}$

25. $16+2\frac{4}{7}-3\frac{1}{3}$

26. An empty soap box weighs $4\frac{1}{4}$ pounds. It is packed with bars of soap weighing $54\frac{1}{8}$ pounds. By adding $4\frac{1}{4}$ and $54\frac{1}{8}$ find the weight of the box after packing.

27. The sum of two numbers is $30\frac{1}{8}$. One of the numbers is $16\frac{3}{4}$. Find the other by subtracting $16\frac{3}{4}$ from $30\frac{1}{8}$.

28. A table top 32 in. wide is to be made by glueing together 4 boards. How wide must be the fourth board if the widths of the others are respectively, $7\frac{7}{8}$ in., $7\frac{3}{4}$ in., and 8 in.?

29. In the drawing (Fig. 20) measure the length and the width of the frame of the picture, each to the nearest sixteenth of an inch. Add the length to the width.

30. In making a drawing of an object architects, designers, surveyors, and draftsmen usually do not draw a full-size picture. Each line in the drawing is made a specified part of the actual length.

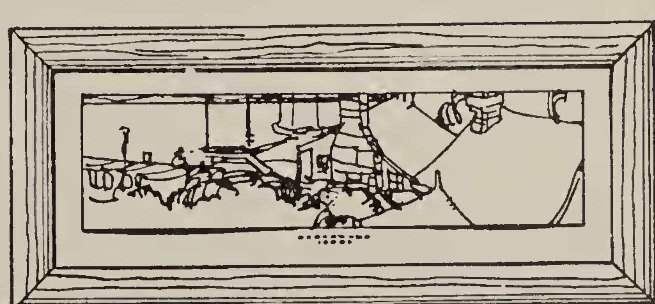


FIG. 20

The object is then said to be *drawn to scale.* The following exercises show how actual length can be found in a scale drawing.

If an inch on the drawing (Fig. 20) represents a length of 8 inches in the frame of the picture, the actual length and width of the frame may be found by multiplying each of the measures in Exercise 29 by 8. Find the actual length and width of the frame.

Fig. 21

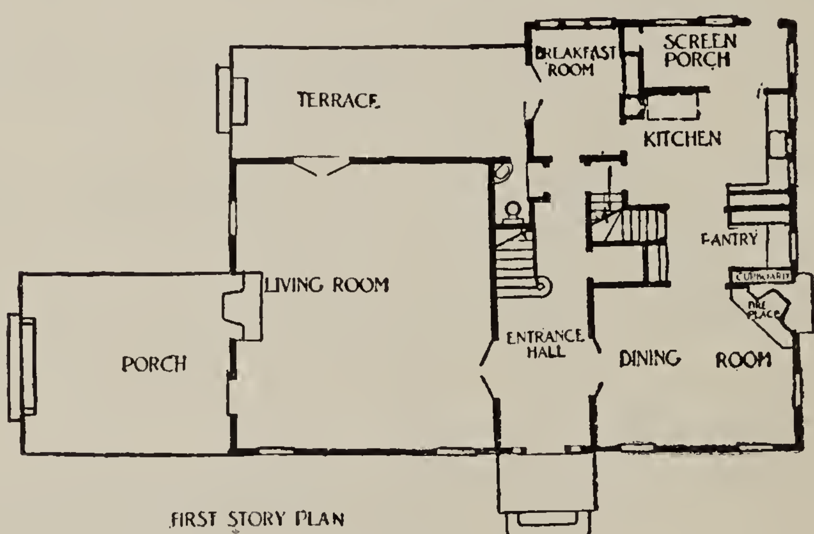


Fig. 22

31. The arrangement of the rooms on the first floor of a house (Fig. 21) is shown in the drawing (Fig. 22). One inch in the drawing represents 24 feet in the house. Find the approximate dimensions of the living room, dining room, kitchen, and porch.

32. Fig. 23 is a map of the State of Illinois. The segment AB in the lower left-hand corner represents a distance of 70 miles. It

Fig. 23

is to be used to measure distances between places on the map to the nearest mile, just as we have used the ruler to measure the distance between the end points of a segment.

Fold a sheet of paper, and on the crease mark off the scale given on AB, making a graduated straightedge.

Using this as a ruler, find to the nearest mile the distance of each of the following cities from Chicago: Rockford, Quincy, Peoria, East St. Louis, Cairo, Danville.

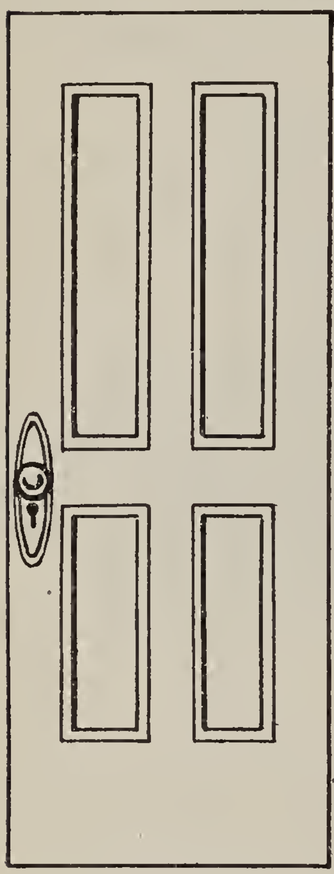


Fig. 24

33. In the drawing (Fig. 24) an inch represents 2 ft., 6 in. Find the dimensions of the door.

Solution: Width $=1 \times 2\frac{1}{2}=2$ ft., 6 inches.

$$\text{Length}=2\tfrac{2}{3}\times 2\tfrac{1}{2}=\frac{8}{3}\times\frac{5}{2}=\frac{\overset{4}{\cancel{8}}\times 5}{3\times\cancel{2}}=\frac{20}{3}=6\tfrac{2}{3}.$$

Length $=6$ ft., 8 inches.

Similarly find the dimensions of the panels.

Exercises 34 and 35 below give practice in such multiplications as occur in Exercise 33.

34. Multiply the following numbers, arranging your work as shown in the first exercise.

$23\frac{1}{6}\times 4.$

Solution: $23\frac{1}{6}\times 4=92\frac{4}{6}=92\frac{2}{3}.$

$\frac{5}{8}\times 7$	$\frac{3}{5}\times 9$	$7\frac{3}{8}\times 16$
$24\frac{2}{3}\times 6$	$\frac{2}{3}\times 15$	$3\times 6\frac{7}{8}$
$6\frac{6}{7}\times 24$	$3\frac{1}{2}\times 5$	$8\frac{2}{3}\times 10$

35. Multiply as indicated the following factors:

$14\frac{2}{3}\times 4\frac{1}{5}.$

Solution: $$14\tfrac{2}{3}\times 4\tfrac{1}{5}=\frac{44}{3}\times\frac{21}{5}=\frac{44\times\overset{7}{\cancel{21}}}{\underset{1}{\cancel{3}}\times 5}=\frac{308}{5}=61\tfrac{3}{5}.$$

The work of writing the solution may be lessened by omitting the step $\frac{44}{3}\times\frac{21}{5}$, but not the step $\frac{44\times\cancel{21}^{7}}{\cancel{3}\times 5}$.

$26\frac{3}{8}\times 12\frac{1}{3}$	$48\frac{3}{4}\times 2\frac{1}{2}$	$1\frac{3}{8}\times 12\frac{1}{2}$
$45\times 33\frac{1}{3}$	$3\frac{1}{4}\times 2\frac{4}{7}$	$16\frac{1}{7}\times 5\frac{1}{4}$
$14\frac{3}{5}\times 11\frac{2}{3}$	$21\frac{2}{3}\times 3\frac{4}{5}$	$14\frac{1}{3}\times 7\frac{2}{5}$
$3\frac{1}{9}\times 12\frac{1}{2}$	$16\frac{1}{4}\times 12\frac{1}{2}$	$9\frac{2}{5}\times 5\frac{5}{6}$
$7\frac{5}{6}\times 8\frac{1}{8}$	$8\frac{2}{7}\times 11\frac{3}{5}$	$18\frac{3}{4}\times 17\frac{1}{3}$
$9\frac{3}{7}\times 13\frac{3}{4}$	$6\frac{4}{9}\times 9\frac{1}{8}$	$24\frac{1}{2}\times 3\frac{1}{8}$

MEASURING LINE SEGMENTS WITH RULER AND COMPASS

8. The compass. So far segments have been measured with a ruler alone. Line segments may be measured with a compass (Fig. 25) as follows:

Open the compass and place the sharp points at A and B, the end points of the segment AB. Then place

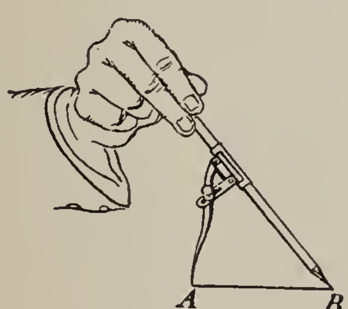


FIG. 25

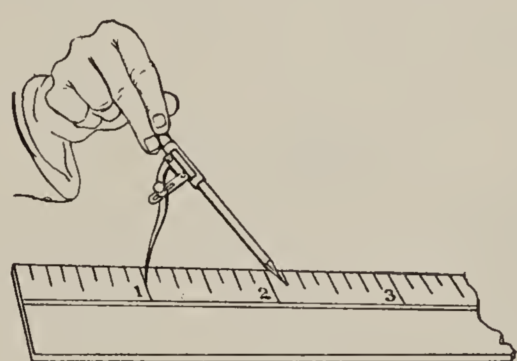


FIG. 26

the points on the marks of a ruler (Fig. 26) and count to the nearest sixteenth of an inch the number of inches between them. This is the *length* of AB. Why?

By this method of measuring with the compass one is able to measure more accurately than with the ruler alone. For, when we measure with the ruler the eye

has to pass from the end points of the segment to the marks on the ruler, which makes it difficult to get the best reading. The error is reduced by carrying with the compass the distance between the end points from segment to ruler. Hence, when *exact* work is required the compass should be used. *It is important to keep the pencil point of the compass sharpened.*

EXERCISES

FIG. 27

1. Using the compass and ruler as shown above measure AB, BC, and CA (Fig. 27) to the nearest sixteenth of an inch.

2. The drawings (Figs. 28 and 29) represent a match safe and the designs (working drawings) for making the safe. A length of an

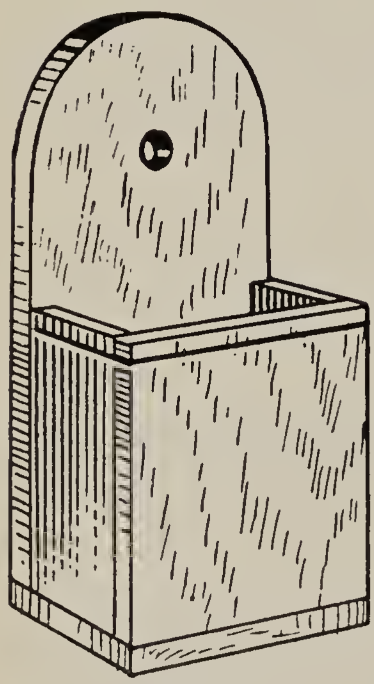


FIG. 28

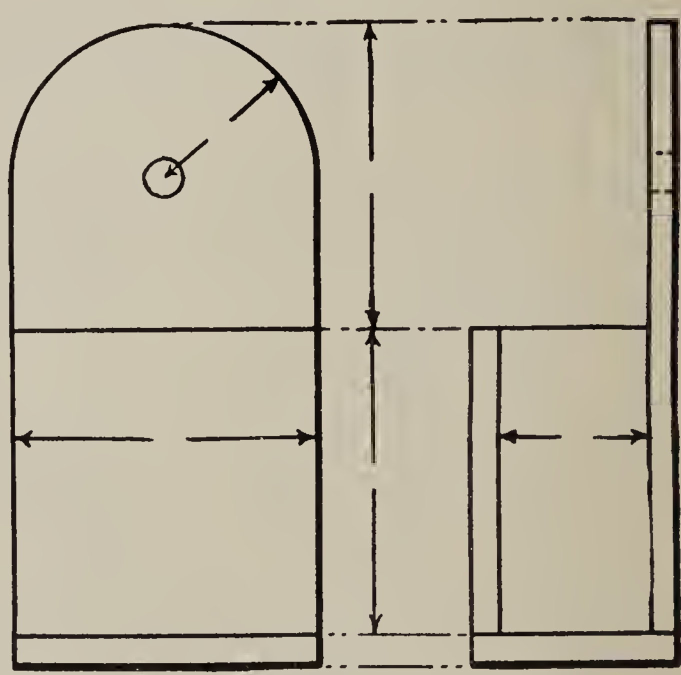


FIG. 29

inch in the design represents 2 inches in the corresponding length of the safe. Find to the nearest sixteenth of an inch the following dimensions of the safe: width, height, depth, and thickness of boards.

9. How standard units of length have been made. Most of the civilized nations have derived a unit of length from the length of the human foot. The result has been that standard units of length are not the same in different countries.

Early units of measurements were indefinite. Thus the *foot-breadth* and *cubit* (distance from the elbow to the extremity of the middle finger) mentioned in the Bible vary for different persons, and exact measurement with these units is impossible. Hence it became necessary for people to adopt a definite unit.

In a book on surveying we find the following account of how people in the sixteenth century tried to obtain a standard unit. "Stand at the door of a church on Sunday, and bid sixteen men to stop, tall ones and small ones, as they happen to pass out when service is finished; then make them put their left feet one behind the other and the length thus obtained shall be a right and lawful rod to measure and survey land with, and the sixteenth part of it shall be a right and lawful foot."[1]

Henry I, King of England (1100–1135 A.D.), is said to have used as a unit the distance from the point of his nose to the end of his thumb, approximately a yard's length. Other units were made in 1490 by Henry VIII and in 1588 by Queen Elizabeth. The present English standard was adopted in 1855.

In 1856 the English Government sent to this country two copies of the new English standard, one made of bronze, the other of iron, which were used as standards

[1] Lessons in Community and National Life. Series B, Chapter VI. By Charles H. Judd and Leon C. Marshall.

until 1875. The table below gives the units of linear measure in that system.

TABLE OF LINEAR MEASURE IN THE ENGLISH SYSTEM

12 inches (in.)	= 1 foot (ft.).
3 feet (ft.)	= 1 yard (yd.).
$5\frac{1}{2}$ yards = $16\frac{1}{2}$ feet	= 1 rod (rd.).
320 rods = 5280 feet	= 1 mile (mi.).
6 feet	= 1 fathom.
1.151 miles an hour	= 1 knot.

During the French Revolution the National Assembly (1790) appointed a committee of the Academy of Sciences to study the matter of finding a suitable system of weights and measures. This commission selected as the standard unit of length one ten-millionth part of the distance from the north pole to the equator, measured along the meridian through Paris. This standard unit is called *meter*. The commission determined the length of a meter to be about 39.37 inches, the work requiring seven years for its completion. This distance was marked off by expert instrument makers on a bar of platinum. On June 22, 1799, the standard unit was presented to the Council of Five Hundred and deposited in the archives at Paris. It is the only unit of length which is the result of scientific investigation. The meter (m.) is divided into 10 equal parts called *decimeters* (dm.). Each decimeter is divided into 10 equal parts called *centimeters* (cm.). Each centimeter is divided into 10 equal parts called *millimeters* (mm.). A thousand meters make a *kilometer* (km.). The following table expresses these lengths in terms of inches, feet, yards, and miles.

Table of Linear Measure in the Metric System

	Inches	*Feet*	*Yards*	*Miles*
Millimeter	0.03937	0.003	0.001	
Centimeter	0.3937	0.033	0.011	
Decimeter	3.937	0.328	0.109	
Meter	39.37	3.281	1.093	
Kilometer	39370.	3280.833	1093.611	0.621

We shall speak of this system of measures as the *metric system.* It is now generally used throughout the world in scientific investigations. In view of the trade between nations it is desirable that they employ a uniform system of measures in commerce. In 1866 a law was passed making the metric system legal in the United States. The system is easier to understand and to use than our common system. Fig. 30 represents

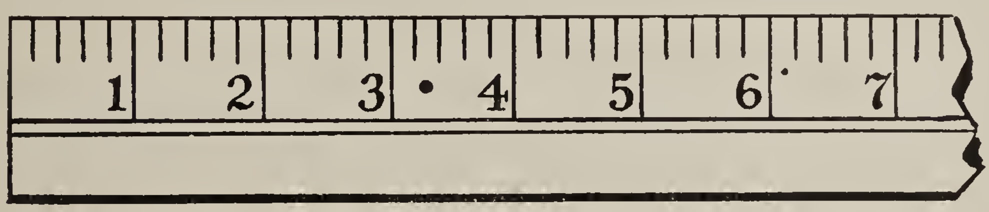


Fig. 30

a part of a ruler graduated according to the metric system.

The following editorial, taken from one of our leading newspapers, describes the merits and importance of the metric system, in urging its adoption:

Adopt the Metric System

One of the bills which ought to be passed by congress is the Britten-Ladd bill, which provides for a gradual adoption of the metric system in the United States. The measure has been well considered and is conservative in its provisions. It provides for a

transition period of ten years in merchandising, leaving manufacturers to adopt the new measures as they please. It is certain that if given a footing metric standardization will approve itself and the existing measures will be abandoned without serious disarrangements or expenditures within a comparatively short time.

The metric system is now used by most modern advanced peoples, ourselves and the British being the important exceptions. It gives easier and in practice more accurate measure than the Anglo-American traditional units of measure, as many an American soldier discovered during the war. But even if it were not more scientific, its adoption would be advisable for us because it is the standard in use in the markets which Americans hope to enter throughout the world. Our present system is a substantial handicap for our foreign trade, in South America for example. Differences in measure of length, capacity, and weight are annoying to our customers and a deterrent to the purchase of American goods.

In proportion as the development of foreign trade is essential to our prosperity the need for accepting the international standard becomes urgent. This is important in peace. In case of war it is even more important. In the late conflict our system of measures was a serious obstacle to prompt and effective coöperation and exchange of resources with our allies. In consequence, men like General Pershing urge adoption of the metric standards. In fact, soldiers, scientists, educators, manufacturers, and commercial men of the highest standing are the emphatic advocates of international standardization upon the metric basis. But the reform is of importance to all of us in proportion as we are all affected, directly or indirectly, by the expansion and efficiency of our trade, both domestic and foreign.

The reform has been on the way too long. Our foreign commerce cannot afford any handicap of which it can rid itself. It will take time to put the system in operation and make the necessary adjustments and it is therefore bad policy to postpone action longer.

Exercises

1. In our reading we find mention of such units as: cubit, pace, hand, ell. Find out what these units mean and how they originated.

2. Using the metric scale on your ruler, measure the length of the page of the textbook to the nearest tenth of a centimeter.

3. Change 268 mm. to centimeters; 3764 mm. to meters; 8 m. to decimeters; 15 m. to decimeters; 2486 mm. to centimeters.

4. The distance between two cities in France is 132 kilometers. How many miles are they apart?

5. The height of an airplane is 800 meters. Express this in feet.

6. The speed of an airplane is 98 kilometers an hour. Express the speed in miles.

7. By means of the compass determine to the nearest sixteenth of an inch the number of inches contained in 3 centimeters; 5 centimeters; 6 centimeters; 8 centimeters. Arrange your results in the form of a table.

8. The distance between two cities in France is 153 kilometers. Express this in miles.

Measuring Line Segments with Squared Paper

10. Squared paper. Fig. 31 represents a part of a sheet of squared paper. It is ruled with east-west and north-south lines. They divide the paper into large and small squares. By measuring with the ruler we find the sides of the large squares to be one centimeter long, the sides of the small squares to be .2 of a centimeter.

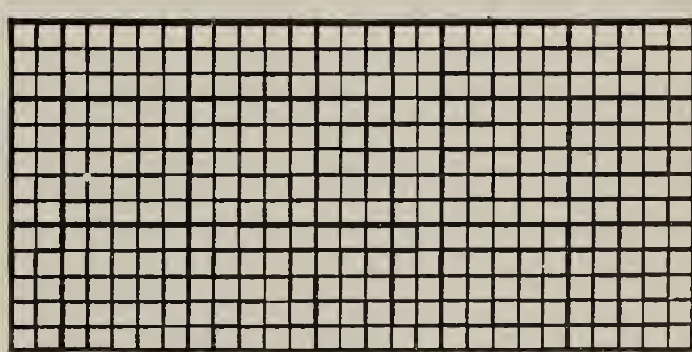


Fig. 31

Thus the lines on the paper are divided according to the *metric system*.

11. How to measure with squared paper. Squared paper may be used to measure segments. It is convenient to select as a unit a segment 2 centimeters long, as AB (Fig. 32).

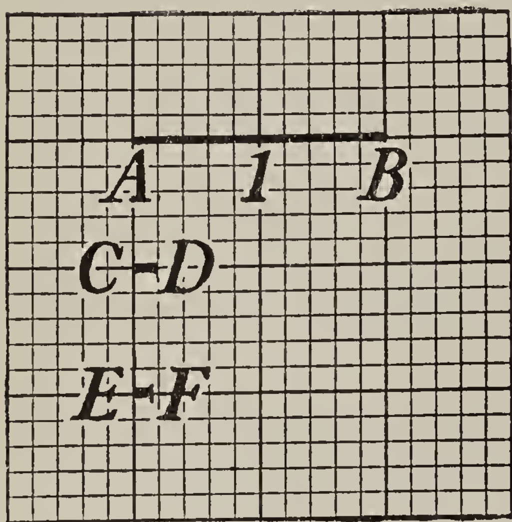


Fig. 32

Then the length of CD is one-tenth of that of AB, or .1.

With a little practice we shall be able to estimate the length of a segment which is shorter than CD. Thus EF is less than CD, but greater than one-half of CD. To determine the length of EF with a fair degree of accuracy *imagine* CD divided into 10 equal parts. Each of these parts is one-tenth of CD, and therefore equal to one-hundredth of AB, or .01. Similarly, one-half of CD is equal to .5 of CD, or .05.

By examining the segment EF we find that it is greater than .05, but less than .1. It seems to be about .08.

Thus, we have seen that if $AB = 1$, then $CD = .1$, and $EF = .08$ approximately. We shall now learn to measure segments, using a unit 2 cm. long.

Exercises

1. Measure AB (Fig. 33).

Directions: Open the compass placing the sharp points at A and B respectively. Place the metal point of the compass on one of the corners of a large square, such as C.

With the pencil point make a mark on one of the heavy lines passing through C. This locates the point D.

Then CD is of the same length as AB, and the length of AB may now be found by measuring CD.

Show that CE is equal to 1.

Show that EF is equal to 1.

Hence CF is equal to 2.

Show that FG is equal to .2.

Show by estimating that GD is .08, approximately.

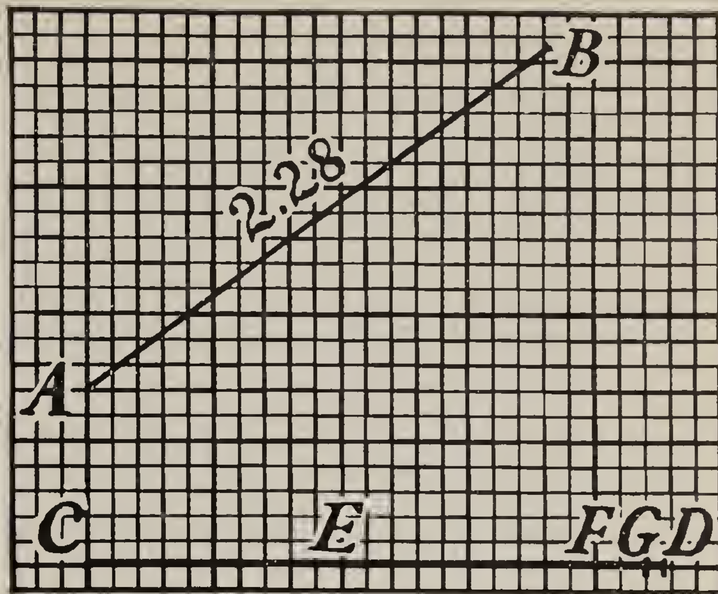


Fig. 33

Show that CD is approximately 2.28.

Hence AB is *approximately* 2.28.

After measuring a segment write the length on the segment, as shown in Fig. 33.

Notice that in this result the first and second figures in the number 2.28 are *exact*, but that the last figure is *uncertain*. We say that AB has been *measured to three figures* or to two decimal places.

2. Show that the length of AB (Fig. 34) is 0.84, when measured to three figures. This result is stated in the form

$AB = 0.84$ approximately.

3. Draw a segment and measure it to three figures. State the result in the form shown in Exercise 2.

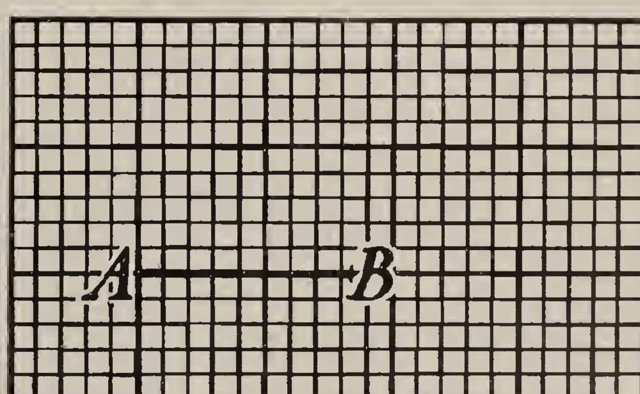


Fig. 34

4. Measure to three figures the segment AB (Fig. 35). Let one pupil write the results of several others on the blackboard. Let each pupil find the average of these results. Which results differ least from the average?

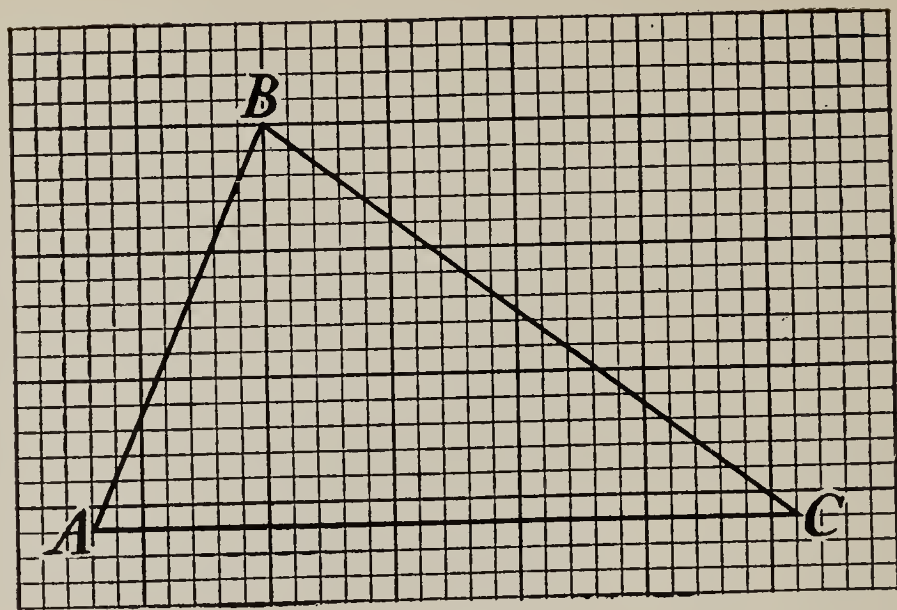


FIG. 35

Measure AC and BC (Fig. 35) each to three figures.

5. Measure to three figures each of the segments AB, BC, CD, and DA (Fig. 36). State the results as in Exercise 2.

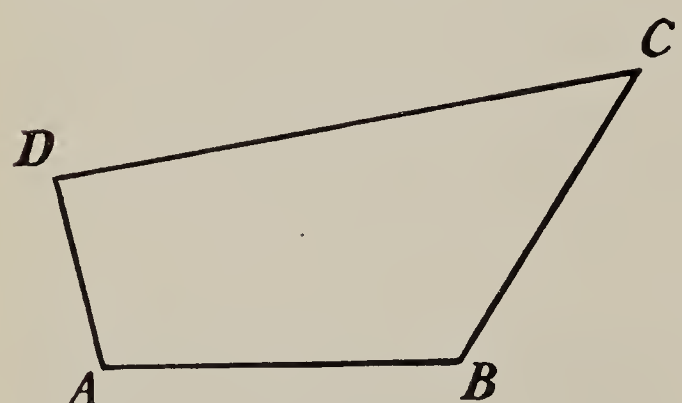


FIG. 36

6. Draw segments AC and DB (Fig. 36), and measure each to three figures.

7. Measure to three figures the distances from A to B (Fig. 37), from B to C, from B to D, from A to C. State the results as in Exercise 2.

12. Symbols of equality and inequality. In §11 we used the statements *is equal to*, *is greater than*, and *is less than*. In mathematics it is convenient and customary to use symbols to denote briefly such verbal statements. Thus,

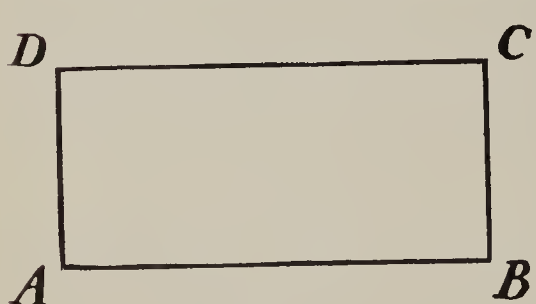


FIG. 37

5 is equal to 4+1 is written $5=4+1$;
6 is less than 8 is written $6<8$;
7 is greater than 5 is written $7>5$.

Thus, $=$ means "is equal to"; $>$ means "is greater than"; $<$ means "is less than."

The pupil must make himself familiar with the meaning of these symbols.

13. Equation. A statement expressing the equality of two numbers, as $5=4+1$, $AB=3.24$, is called an **equation.**

Exercises

Write each of the following statements in symbols: five is greater than three; eight is equal to the sum of six and two; seven is less than ten; AB is less than MN; x is greater than y; the difference between a and b is less than the sum of a and b.

14. Notation for line segments. We have denoted line segments by marking the end points with *capital* letters. Sometimes one *small* letter is used to denote a line segment. This is written on the segment near the midpoint, as a (Fig. 38).

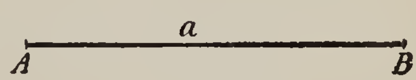


FIG. 38

Capital letters usually represent *points*, *small* letters denote *numbers*. In Fig. 38 the number a is an *unknown* number. It denotes the *length* of the segment, and may be found by measuring the segment.

Exercises

1. Measure AB (Fig. 38), and thus find the number denoted by the small letter a.

2. By measuring find the numbers denoted by a, b, and c (Fig. 39).

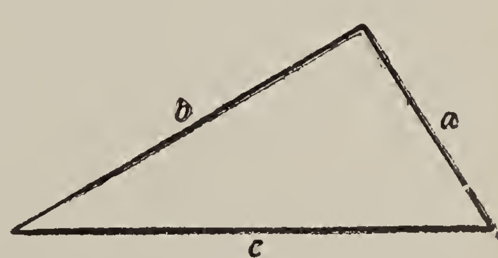


FIG. 39

15. Literal number. There are various ways of denoting numbers by symbols. For example, the number five may be denoted as follows: 5, V, ⁙, or 卌. A number denoted by a letter is a **literal number.**

Comparing Lengths of Segments by Means of Ratios

16. How to find the ratio of line segments. Let AB and CD be two line segments whose lengths are respectively 2.38 and 3.15, and let it be required to compare the length of AB with that of CD.

We have $AB = 2.38$ in.
and $CD = 3.15$ in.

Dividing the length of AB by that of CD, we have the quotient

$$\frac{AB}{CD} = \frac{2.38}{3.15} = 0.75$$

or $$\frac{AB}{CD} = 0.75$$

Computation:

$$\begin{array}{r} 0.755 \\ 3.15\overline{)2.380} \\ 2\,205 \\ \hline 1750 \\ 1575 \\ \hline 1750 \\ 1575 \\ \hline \end{array}$$

This result is not *exactly* equal to $\frac{2.38}{3.15}$, since we have dropped all figures after the second decimal place. However, it is true *approximately to the third figure*, or to *two decimal places*.

The quotient .75 is the *ratio* of AB to CD. This ratio compares segment AB with segment CD. It indicates that AB is about .75 of CD or $\frac{3}{4}$ as long as CD.

In general, the **ratio** of two segments is the quotient found by dividing the measure of one by that of the other, provided a common unit is used in measuring.

Thus, if one segment is 2 in. long and another 3 in., the *ratio* of the segments is $\frac{2}{3}$, and the length of the first segment is $\frac{2}{3}$ of the length of the second.

Exercises

1. Draw two segments, AB and CD. Measure the segments to three figures. Find the ratio by dividing one measure by the other. Arrange your work as shown in §16. State your results approximately to two decimal places.

2. If $AB=3.16$ and $CD=1.24$, find the ratio to three figures, arranging your work as in the example of §16.

3. In triangle ABC (Fig. 40) measure each of the segments AB, BC, and CA to three figures and find, to the third figure, the ratios

$$\frac{a}{b},\ \frac{b}{c},\ \frac{c}{a},\ \frac{b}{a},\ \frac{c}{b},\ \frac{a}{c}.$$

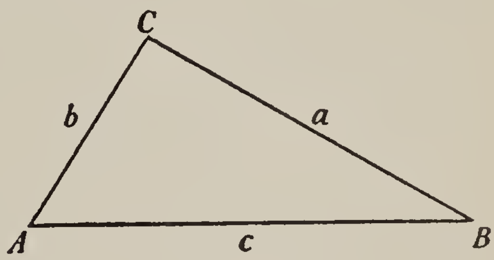


Fig. 40

4. Divide to three figures as indicated:

264 by 681	1.03 by 6.17	23.8 by 24.1
301 by 126	0.15 by 8.34	1.40 by 915
615 by 305	9.00 by 0.02	0.12 by 218

17. How to find the ratio of two numbers. We have seen that lengths may be compared by means of ratios. Two *numbers* may be *compared* with each other by dividing. Thus, to compare 8 with 12 we divide 8 by 12. The comparison is then expressed by the ratio $\frac{8}{12}$ or $\frac{2}{3}$, meaning that 8 is $\frac{2}{3}$ of 12.

In the following table compare each number in the first line with the corresponding number in the second by dividing the number in the first by the corresponding number in the second.

6 in.	2 in.	8 lb.	4 lb.	18 hr.	\$16
2 in.	6 in.	4 lb.	8 lb.	6 hr.	\$ 4

The *quotient* found by dividing 6 by 2 is the *ratio* of 6 to 2. In general, the *ratio* of a number to another is the quotient obtained by dividing. The ratios of 6 to 3, and of a to b, may be written in the form $\frac{6}{3}$, $\frac{a}{b}$, and read "6 over 3," "a over b," meaning 6 divided by 3, a divided by b. In arithmetic $\frac{6}{3}$ is usually read "6 thirds."

Exercises

Express the ratios of the following, reducing each to the lowest terms.

1. 42 to 56.

Solution: Ratio $=\frac{42}{56}$.

Dividing numerator and denominator first by 2 and then by 7, we have

$$\text{Ratio}=\frac{\overset{\overset{3}{\cancel{21}}}{\cancel{42}}}{\underset{\underset{4}{\cancel{28}}}{\cancel{56}}}=\frac{3}{4}.$$

2. 4 to 9.
3. 8 to 18.
4. 18 to 33.
5. 12 to 15.
6. 198 to 12.
7. 24 to 63.
8. 3 to a.
9. y to x.
10. d to t.
11. $\frac{7}{8}$ to $\frac{3}{4}$.
12. $\frac{2}{3}$ to $\frac{5}{6}$.
13. $\frac{36}{40}$ to $\frac{27}{35}$.

Reduce the following fractions:

14. $\frac{60}{90}$
15. $\frac{33}{44}$
16. $\frac{21}{28}$
17. $\frac{72}{81}$
18. $\frac{125}{100}$
19. $\frac{35}{40}$
20. $\frac{56}{124}$
21. $\frac{76}{132}$
22. $\frac{63}{846}$

Measuring by Hundredths

18. Expressing length as hundredths of a unit. On squared paper draw a segment 10 cm. long (one decimeter). In the following this is to be used as a

unit of measurement (Fig. 41). Imagine each centimeter divided into 10 equal parts (millimeters) as on the centimeter ruler.

Each millimeter is $\frac{1}{10}$ of a centimeter, or $\frac{1}{100}$ of the unit ($\frac{1}{100}$ of a decimeter).

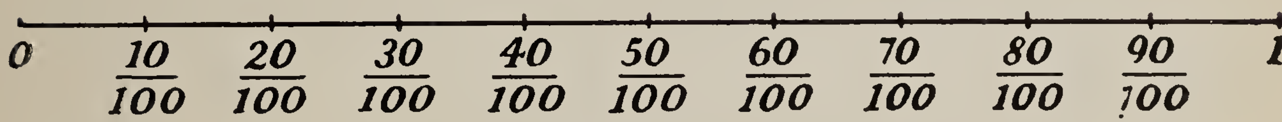


Fig. 41

Each centimeter is therefore $\frac{10}{100}$ of the unit.

We shall now see how to measure line segments to the nearest one-hundredth part of the unit.

Exercises

1. Show from a drawing that a segment equal to $\frac{1}{2}$ of a decimeter is equal to $\frac{50}{100}$ of it.

2. Using a decimeter as a unit, express the following line segments as hundredths.

$\frac{1}{5}$ of a decimeter = ———— hundredths $= \frac{\quad}{100}$
$\frac{3}{4}$ of a decimeter = ———— hundredths $= \frac{\quad}{100}$
$\frac{4}{5}$ of a decimeter = ———— hundredths $= \frac{\quad}{100}$
$\frac{1}{3}$ of a decimeter = ———— hundredths $= \frac{\quad}{100}$

19. Meaning of per cent. In Exercise 2 hundredths were expressed by means of fractions having 100 as denominator. The words *per cent,* meaning hundredths, are also used. The sign %, read *per cent,* has the same meaning.

Exercises

1. The results of Exercise 2 may be expressed as follows:
$\frac{1}{5} = \frac{20}{100} = 20$ hundredths $= 20$ per cent $= 20\%$.
$\frac{3}{4} = \frac{75}{100} = 75$ hundredths $= 75$ per cent $= 75\%$, etc.
Express similarly the equivalents of $\frac{4}{5}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{3}{5}$.

2. If part of a unit is 5%, or $\frac{5}{100}$, of the unit, what per cent is the remaining part?

3. Draw a line segment 1 dm. long. Mark off 12%; 45%.

4. Draw a line segment of any length. Mark off 25% of it; 15% of it.

5. Express the following per cents as hundredths: 3%; 8%; 25%; 60%.

6. Make a drawing to show 10% of 2.

Draw a segment 2 decimeters long as shown in the scale drawing below (Fig. 42). Since 10% of a decimeter is equal to $\frac{10}{100}$ of it, show that

$$\frac{10}{100} \text{ of } 1+\frac{10}{100} \text{ of } 1 = \frac{10}{100} \text{ of } 2.$$

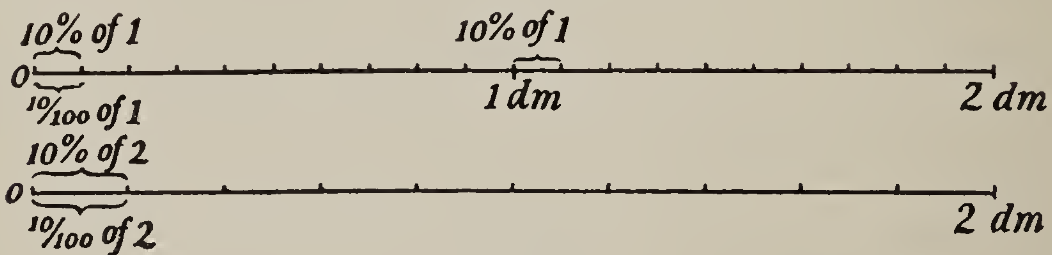


FIG. 42

7. Exercise 6 shows that 10% of 2 means $\frac{10}{100}$ of 2, or $\frac{10}{100}\times 2$. State the meaning of each of the following:

8% of 4; 6% of 8; 3% of 7.

8. Find 5% of 40.

Solution 1: 5% of $40=\frac{5}{100}$ of $40=\frac{5}{100}\times 40=\frac{\cancel{5}\times\overset{2}{\cancel{40}}}{\underset{\cancel{5}}{\cancel{100}}}=2.$

Solution 2: 5% of $40=.05\times 40=2.$

9. Find 5% of 60; 10% of 80; 25% of 200.

20. Uses of per cent in daily life. Measurement by hundredths is used widely in business, in scientific work, and in statistical reports as shown by the following statements:

A man saves 10% of his salary.

A merchant makes a profit of 20% on his sales.
The price of a suit is reduced 15%.
Milk tests 4% butter fat.
The sales increased 10%.
The bank pays 3% for the use of money.
Tell what each of these statements means.

21. What every pupil should know and be able to do. In the preceding pages the meaning of line segment has been made clear through measuring and drawing.

Below is a list of terms and facts which every pupil should understand.

1. The meaning and correct use of the terms: Line, point, segment, unit segment, ratio of two segments, scale drawing, metric system, exact and approximate measure, literal number, ratio of numbers.

2. The meaning of the following principles:

a. Through one point any number of straight lines may be drawn.

b. Only one straight line can be drawn through two points.

c. Two intersecting straight lines can have only one point in common.

3. Understanding of what *per cent* means.

4. The tables of linear measure in the English and metric systems.

22. Typical problems and exercises. The pupil should be able to use the ruler, squared paper, and compass in measuring and drawing line segments; to

add, subtract, multiply, and divide accurately common and decimal fractions; and to write a satisfactory test paper on questions and problems of the type given below.

1. Draw a line segment and find the length using only a ruler; using compass and ruler; using compass and squared paper.

2. Draw a segment and measure it to three figures; to the nearest sixteenth of an inch.

3. Explain the metric system of measuring lengths. How are meters changed to centimeters? Centimeters to decimeters?

4. State the meaning of the following symbols: $=$, $>$, $<$.

5. Add and subtract as indicated: $18\frac{1}{2}+10\frac{1}{4}-15\frac{3}{8}$.

6. Draw a line segment. Mark off 15% of it.

7. Multiply $3\frac{1}{2}\times2\frac{4}{7}\times8\frac{2}{3}$.

8. Find the ratio $\frac{2.38}{3.15}$ to three figures.

9. Find the average of the following measures:
3.64, 3.59, 3.60, 3.61, 3.63, 3.62.

10. The dimensions of a room are 24 ft. by 18 ft. Make a drawing of the room representing 10 ft. by one inch.

11. Reduce the ratio $\frac{210}{120}$ to lowest terms.

12. Write a paper on one of the following topics:
a. The need of measurement in daily life.
b. How standard units of measurement have come into use.
c. Scale drawings.
d. The metric system.

CHAPTER II

HOW WE USE LINE SEGMENTS IN PICTURING NUMERICAL FACTS

Representing Numerical Facts by Graphs and Tables

23. Picture-representation of numerical facts. Newspapers and magazines often represent numerical statistics, scientific data, or facts by the use of drawings and pictures. Such pictures are known as *pictograms*, or *picture graphs*. Thus, the picture graph (Fig. 43) illustrates the amounts of the leading crops in the United States in 1923, each being pictured by the number of bags on the truck. The differences in the crops are easily seen by comparing the various loads as to size.

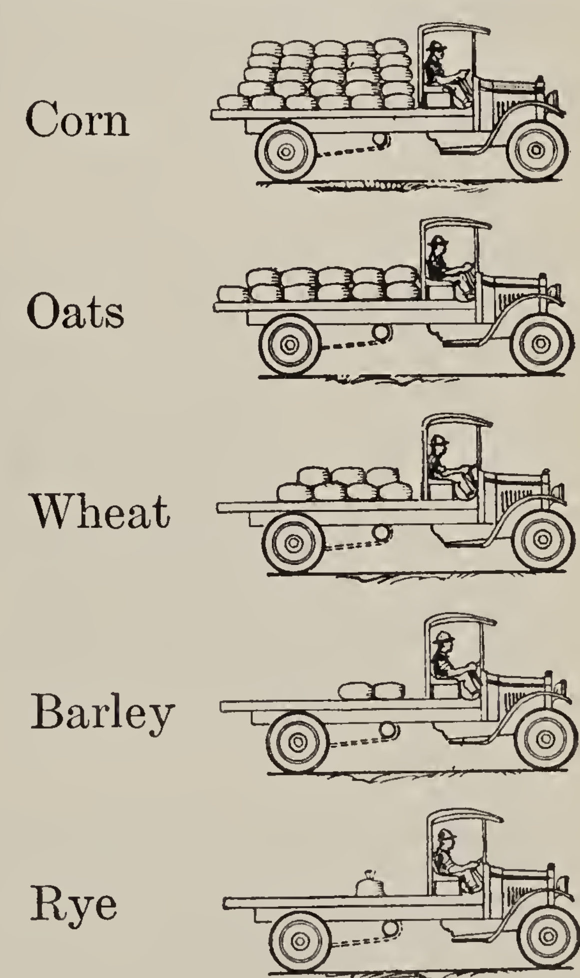


Fig. 43

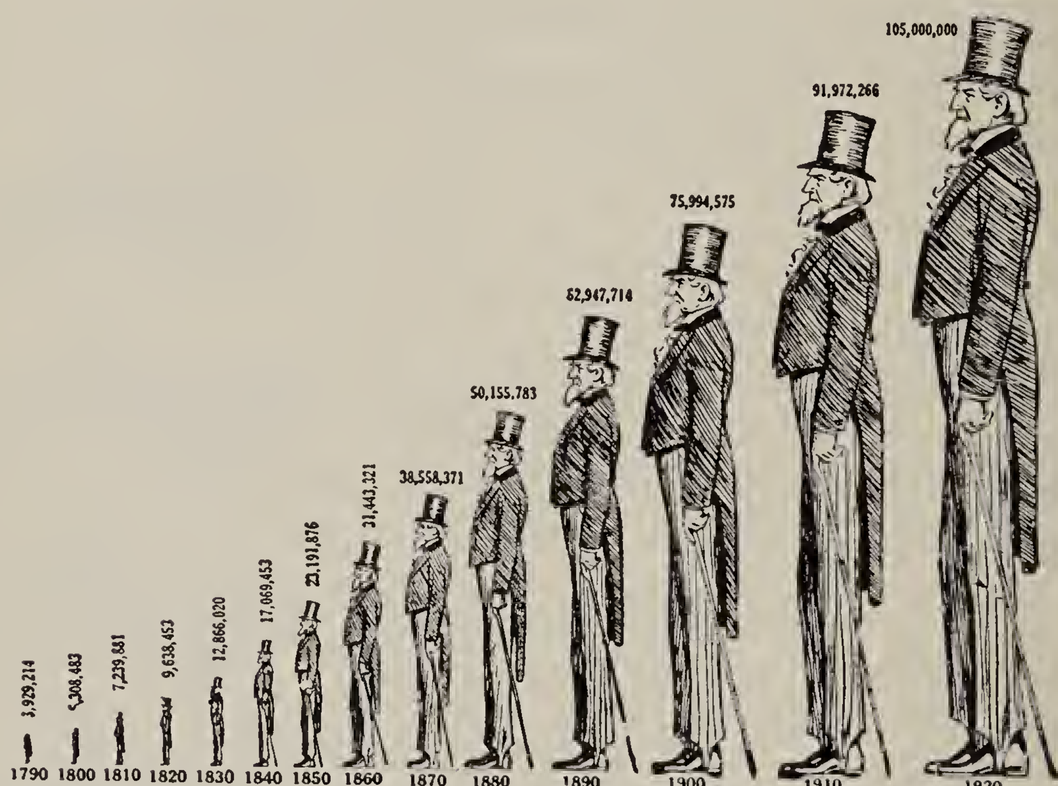
The same facts might be stated in the form of the table on page 36.

Report of the United States Department of Agriculture for 1923

Corn	*Oats*	*Wheat*	*Barley*	*Rye*
3,075,786,000	1,311,687,000	789,227,000	199,337,000	64,800,000

The table should be read as follows: In 1923 the United States raised 3,075,786,000 bushels of corn, etc.

In the picture graph (Fig. 44) the *height* of each figure represents a number of millions. It shows how



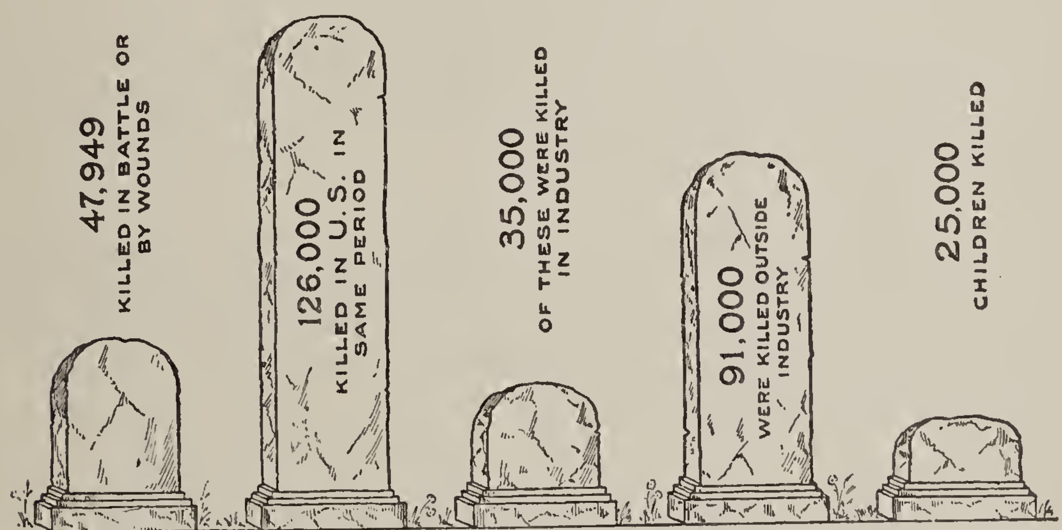
Courtesy of the Literary Digest, N. Y.

Fig. 44

our continental population kept up a steady rate of increase from one decade to the next. However, in 1920 the population was only 105 millions, while it would have been about 111 millions if it had maintained the rate of increase of the previous decades.

In some cases when the picture graph is used to represent numerical facts, there is a danger of forming wrong impressions. In comparing data represented by two-dimensional figures when only length is to be considered, the surfaces covered may not have the same relations to one another as the lengths. Thus, the *surface* of the 1920 figure is 4 times as great as that of the 1880 figure, while the *length* of one is only 2 times as great as that of the other. Hence, a comparison of the surfaces would lead us to believe that the population of 1920 was 4 times as great as that of 1880, which is not in keeping with the fact.

In Fig. 45 we find a pictogram illustrating the fatalities at home while this country was at war.



Courtesy of the Literary Digest, N. Y.

FIG. 45

The objection raised against Fig. 44 does not apply to the diagram (Fig. 45), for all the stones have the same width, which makes it possible to compare the surfaces in the same way as the heights. In Fig. 45, not only the heights of the stones but also the surfaces represent the number killed.

The picture method is preferred to the tabular method mainly because the meaning of numerical facts is more easily seen and understood from pictures than from tables. From a row of figures it is not easy to grasp the real facts. For figures must be studied thoroughly, but a picture of numerical facts can be understood without difficulty. Pictures present to the mind in a definite, clear, and comprehensive manner the relations between facts. It is therefore important that readers of magazines and newspapers learn how to interpret numerical facts and relations of facts when they are stated in graphical form.

Several other types of graphical representation may be used for certain kinds of facts. Some of them may not give an impression as clear as that gained by a picture graph, but they have other advantages, such as being easily made and being very accurate. These types are to be discussed in §§24 to 27.

24. How to represent numbers by segments. Draw a line segment 3 centimeters long. This segment is said to *represent* the number 3. In the following exercises we shall learn how to represent numbers by segments.

Exercises

1. Draw a segment 5 cm. long. What number does this segment represent?

2. Draw a segment $3\frac{1}{2}$ in. long. What number is represented by the segment?

3. For each of the following numbers find the unit segment most convenient for representing the number by a segment. Then draw the segments representing the numbers

$$4.5,\ 6,\ 2\tfrac{5}{8},\ 1\tfrac{3}{4},\ 3\tfrac{4}{5}.$$

4. Represent the number 2.38 by a segment.

Directions: As unit select a segment equal to 2 cm.

Draw a segment of convenient length, as AB (Fig. 46).

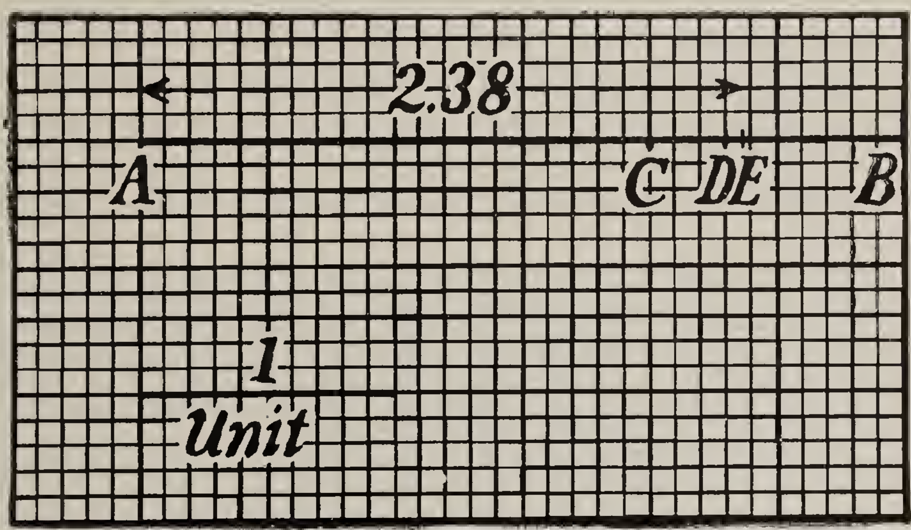


Fig. 46

On AB lay off the unit twice, extending from A to C. AC is then equal to 2 and represents the first digit of 2.38.

To represent the second digit, lay off from C in the direction CB a segment CD three times as long as the side of a small square. The length of CD is .3.

Finally, starting at D lay off DE in the direction DB by *estimating* .8 of the side of a small square.

Show that AE represents the number 2.38.

5. Represent, as in Exercise 4, the following numbers by segments: 1.55, 2.25, 1.37, 1.42.

25. What graphical representation means. When a segment is used to represent a number it is called a *graph* of the number, and it is said to represent the number *graphically*.

26. How to make bar graphs. The table below states the number of hours electric light is used in the average residence in a certain city for each month of one year.

Months	*Jul.*	*Aug.*	*Sep.*	*Oct.*	*Nov.*	*Dec.*	*Jan.*	*Feb.*	*Mar.*	*Apr.*	*May*	*June*
Average daily hours	1.7	2.2	3	5	6.1	6.8	6.5	5.3	4.2	2.5	1.9	1.5

The table should be read as follows: In July the average family burns electric light for 1.7 hours, etc.

To represent these facts graphically, the months in the table may be marked off horizontally (Fig. 47) on

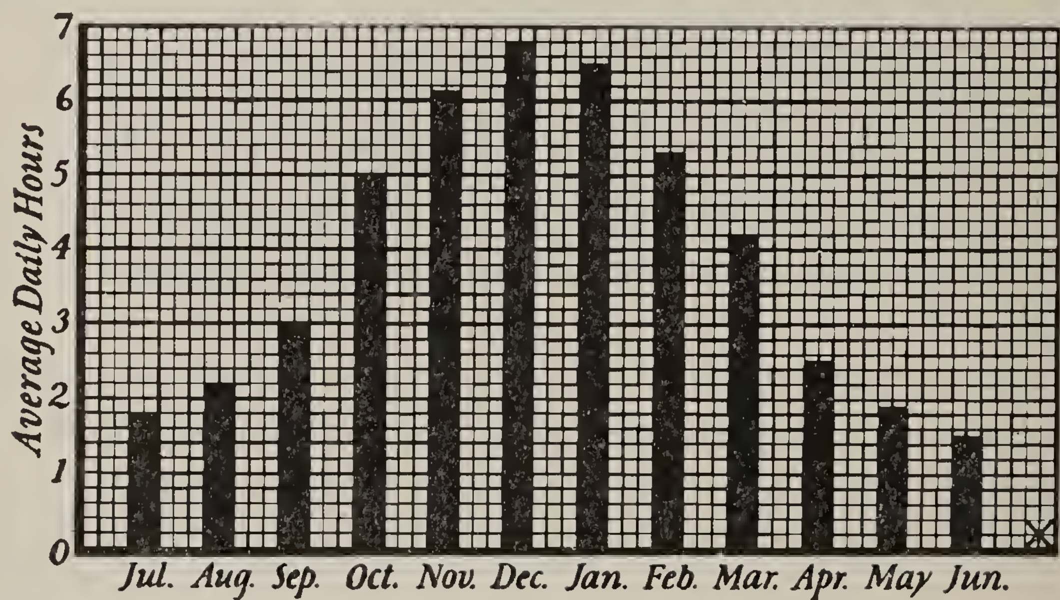


FIG. 47. BAR DIAGRAM SHOWING THE AVERAGE DAILY HOURS ELECTRIC LIGHT IS USED IN THE AVERAGE RESIDENCE.

the segment OX, and the number of hours may be represented by vertical segments drawn upward from the points on OX which represents the various months.

This method of stating facts graphically not only supplements the tabular representation but has some advantages over it. For example, all the facts given in the table above can be seen almost with one glance at the diagram.

Furthermore, the graph shows clearly how much more light is used in December than during any other month; that in June the amount of light used is least; that in winter light is used approximately four times

as many hours as in summer. The graph illustrates how the amount of light used depends upon the time of the year, and explains how our electric light bills *vary* (change) from one month to the next.

The type of graph used in Fig. 47 is called a **bar graph.** The bar graph is easily constructed and understood. The *length* of the bar shows the *size* of the number.

Exercises

1. Figs. 48 and 49 are bar graphs. The first line (Fig. 48) means that in 1922 in China the population was 318 million. Explain the meaning of each of the others. Make comparisons for the different countries by means of ratios.

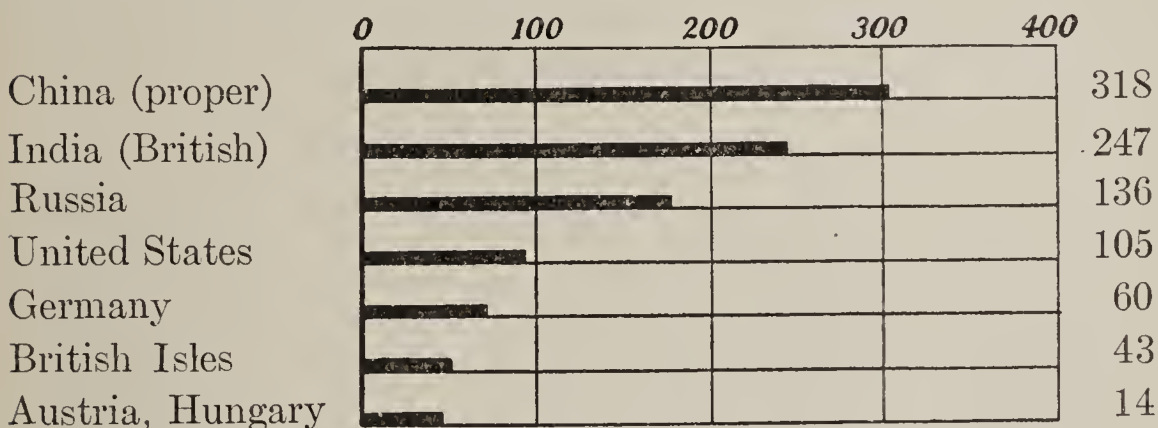


Fig. 48. Bar Diagram Showing Population, in Millions, of Leading Countries in 1922.

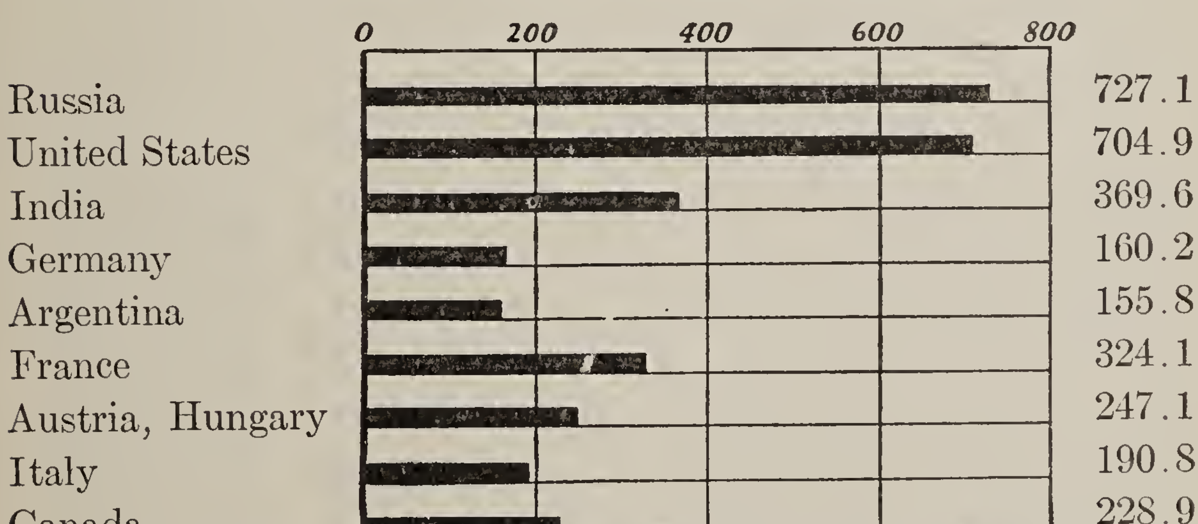


Fig. 49. Bar Diagram Showing the World Wheat Crop in Millions of Bushels. Average for 4 Years.

2. As in Exercise 1, explain the meaning of Fig. 50 and make comparisons.

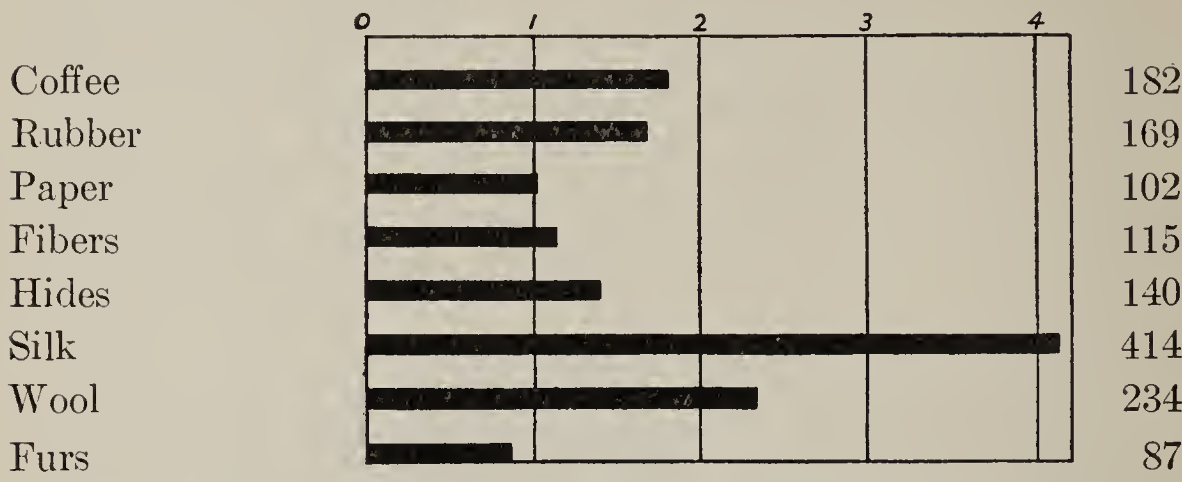


FIG. 50. DIAGRAM SHOWING LEADING IMPORTS INTO THE UNITED STATES FOR THE YEAR 1922–23 IN MILLIONS OF DOLLARS.

3. Using 1,000,000 as a unit, make a bar graph to illustrate the areas of the continents given in the table below.

AREAS OF CONTINENTS

Continents	*Areas in Square Miles*	*Continents*	*Areas in Square Miles*
North America . . .	8,000,000	Asia	17,000,000
South America	6,850,000	Africa	11,000,000
Europe	3,800,000	Australia	3,000,000

4. Represent by means of a bar graph the following table:

EXPORTS OF DOMESTIC MERCHANDISE FROM THE UNITED STATES DURING THE YEAR 1922–23

Cotton	$658,982,855
Grains	451,341,734
Iron and Steel	199,848,561
Cotton Goods	145,360,208
Oils	144,220,168
Meats	143,291,899
Coal	138,215,110
Wood and Manufactures	119,772,940
Copper and Manufactures	113,397,128

5. In the adjoining graph (Fig. 51) the whole bars represent income from sales; the white parts represent profits of a certain company; the shaded parts represent costs. Give the approximate cost, profit, and sale for each year.

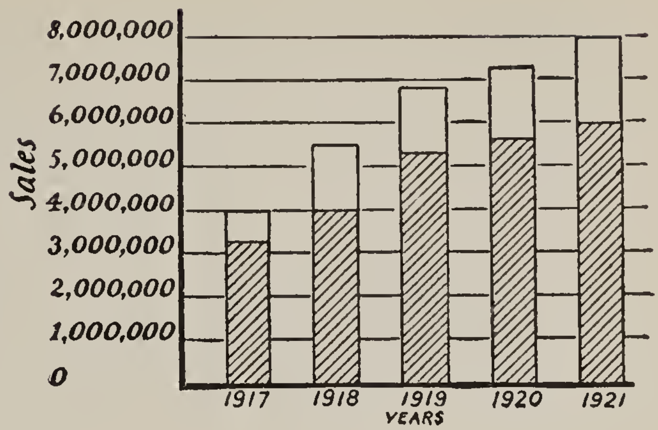


Fig. 51

Using 1,000,000 as a unit, *i.e.*, using only the figures 2.8, 1.2, etc., draw bar graphs for Tables 6 and 7 below:

Suggestion: Use the scale 2 cm. = 1,000,000.

The Principal Crops of the United States for 1922 and 1923

Crops in Thousands of Bushels	6. 1922	7. 1923
Corn	2,890,712	3,075,786
Oats	1,251,496	1,311,687
Wheat	856,211	789,227
Potatoes	451,185	389,674
Barley	186,118	199,337
Sweet potatoes	109,543	93,500
Rye	95,497	64,800
Rice (rough)	41,965	32,600
Buckwheat	15,050	13,500
Flaxseed	12,238	19,407

8. Represent by means of a bar graph the following table stating the cotton production in the United States for the years 1917 to 1922:

Years	*Bales*	*Years*	*Bales*
1917	11,302,000	1920	13,439,603
1918	12,041,000	1921	7,953,641
1919	11,420,763	1922	9,762,069

9. The table below gives the origin of our foreign-born white population according to the Census of 1920. Using 1,000,000 as a unit, make a bar graph of the numbers in the table, *i.e.*, represent graphically the numbers 2.17, 1.69, etc.

Mother Tongue	*Number*
English and Celtic*	2,171,694
German	1,686,102
Italian	1,610,109
Russian	1,400,489
Canadian	1,117,878
Swedish	625,580
Austrian	575,625
Mexican	478,383
Hungarian	397,282
Norwegian	363,862
Total 10 mother tongues	10,427,004
Other mother tongues	3,285,750
All mother tongues	13,712,754

10. A grocery order retailing for $22.00 in 1917 sold for $24.50 in 1918, for $36.00 in 1919, for $50.00 in 1920, and for $18.50 in 1921. Make a bar graph of these facts, showing how food prices changed from 1917 to 1921.

27. How to make line graphs. The diagram (Fig. 52) pictures the temperature readings taken each hour on a certain day. By joining the top points by a continuous line the changes in temperature are exhibited in a more satisfactory way than they would be if the bars alone were used. In fact, if the bars were omitted entirely, leaving only the line passing through the top points, this line would picture the changes in temperature and the hourly readings.

*Includes persons reported as Irish, Scotch, or Welsh.

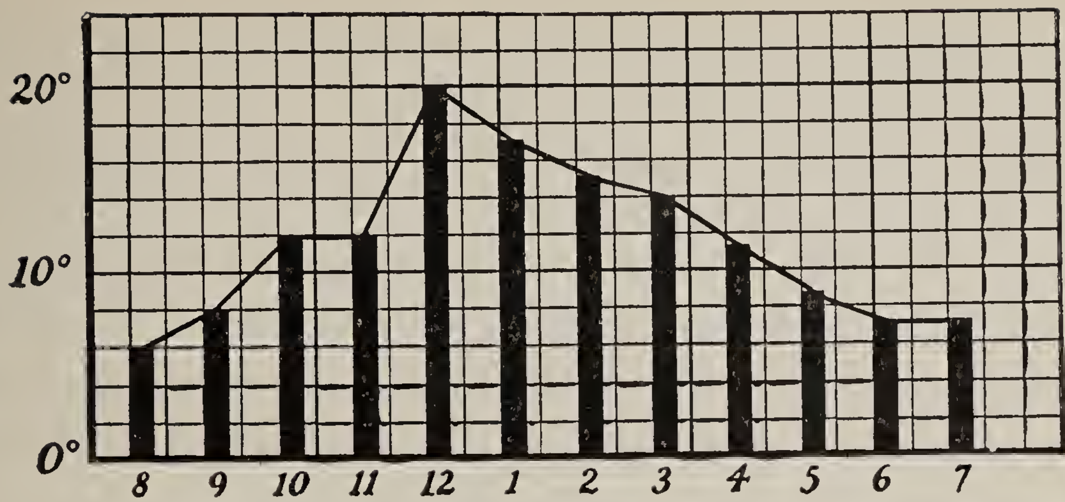


Fig. 52

The average heights of boys and girls, given in the table below, are represented graphically in Fig. 53. The vertical bars are omitted in the graph and only the top points are marked. For when squared paper is used the vertical lines on the paper make the heavy bars unnecessary.

Ages	*Boys*	*Girls*
2 years	2.7 ft.	2.7 ft.
4	3.2	3.2
6	3.7	3.5
8	3.9	3.8
10	4.4	4.2
12	4.5	4.6
14	4.9	5.0
16	5.4	5.1
18	5.6	5.1
20	5.7	5.4

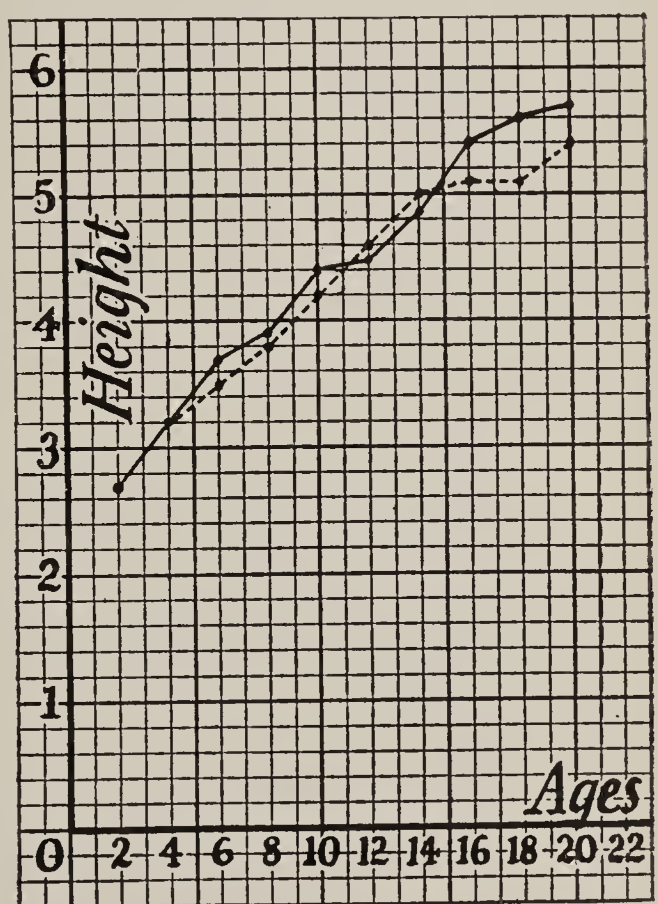


Fig. 53

By joining the top-points with lines the changes in growth may be brought out even better than by

drawing only the isolated facts given in the table. This type of graph is called a **line graph.**

State some facts shown more clearly in the graph than in the table.

The following table of statistics gives the population of the United States from 1800 to 1920.

Year	*Population*
1800	5,308,483
1810	7,239,881
1820	9,638,463
1830	12,860,702
1840	17,063,353
1850	23,191,876
1860	31,443,321
1870	38,558,371
1880	50,155,783
1890	62,947,714
1900	75,994,575
1910	91,972,266
1920	105,710,620

The graph in Fig. 54 represents the same facts. It is a combination of the bar graph and line graph.

Compare this graph with the picture graph in Fig. 44.

The two types of graphs shown in §§26 and 27 are used for different purposes. The *bar graph* should be used to picture *unrelated* facts, as the production of a certain article by various nations, the wealth of several countries. The *line graph* not only illustrates given facts and enables us to make comparisons, but makes it pos-

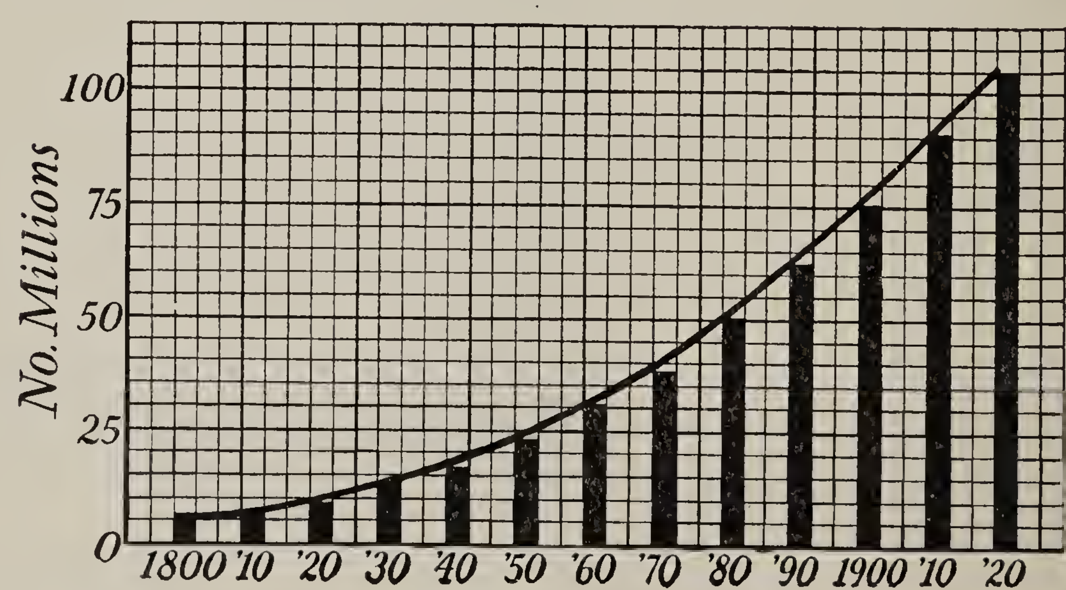


Fig. 54

sible to derive additional facts that are not actually stated in the table or drawn in the diagram. For example, in Fig. 54, we may estimate the population in 1895, or in 1855; we may even predict what under ordinary conditions should be the population in 1930. *Changing* prices of a commodity, temperatures, stock fluctuations, cost relations, are best pictured by *line* graphs.

Exercises

1. The graph (Fig. 55) shows the hourly variation of telephone calls in a great city. Study the graph and answer the following

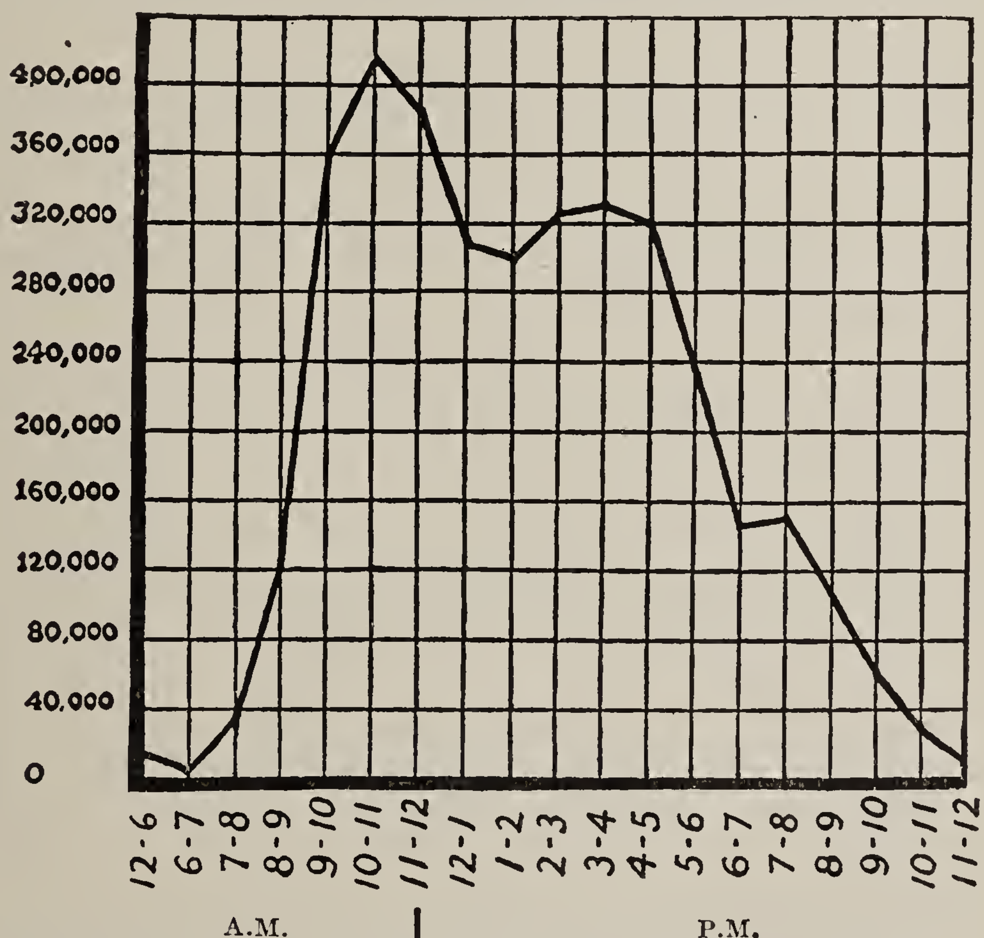


Fig. 55. One Day of New York's Telephone Traffic.

questions. When do people use the telephone most often? How do the calls between nine and ten compare with the calls between eight and nine? How do you account for the drop between twelve and two?

2. The graph (Fig. 56) gives the changes in the price of wheat for one year. What was the price on the first day of each month? When was it lowest? When was it highest? What is your explana-

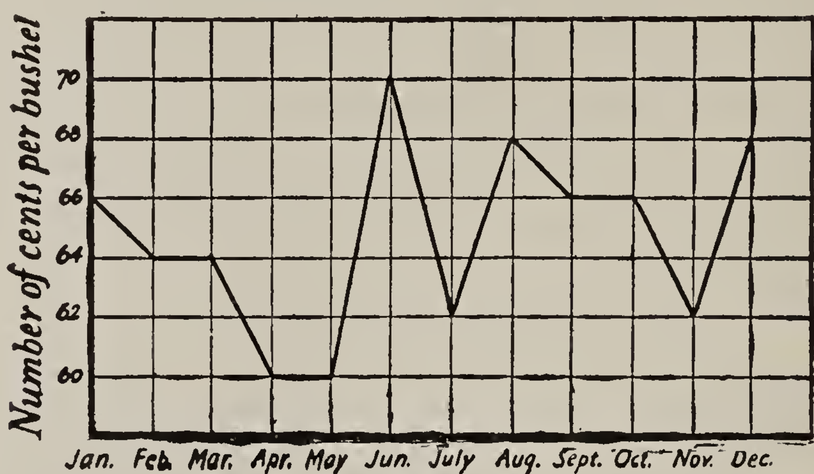


Fig. 56

tion of the last two answers? In which month was the advance greatest? When was the drop greatest? When was the price 65¢?

3. Study the graph (Fig. 57). Find out why immigration was low in 1862, 1896 and high in 1883, 1907. How do you explain the drop after 1907?

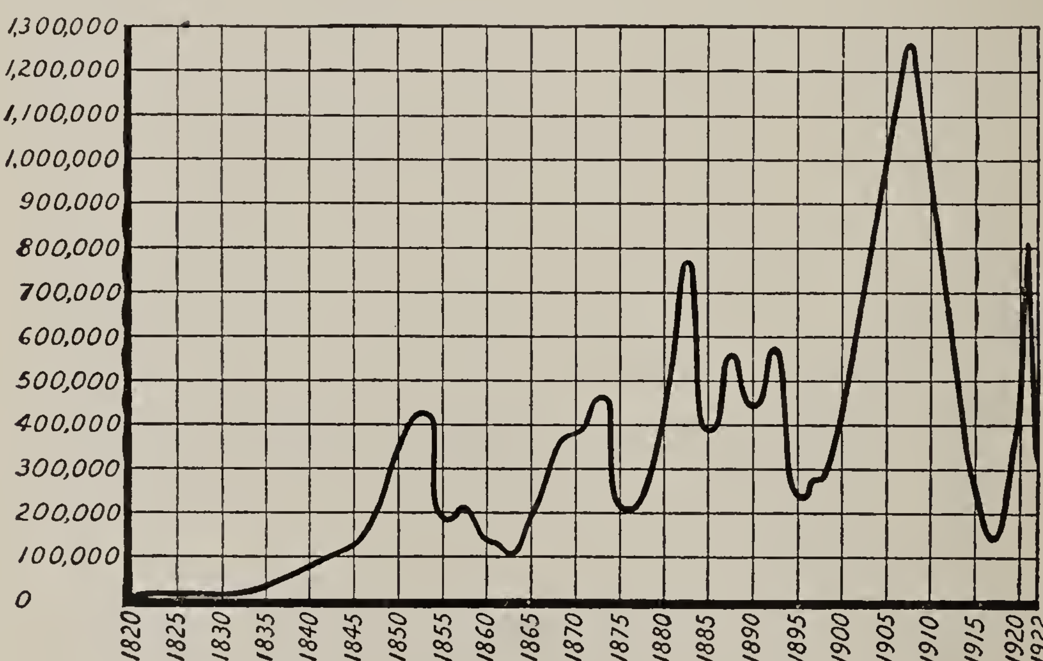


Fig. 57. The Rise and Fall of Immigration.

4. Make a graph of imports of raw cotton from the United States to Great Britain for 1918 to 1922: 9,760,182; 13,707,407; 13,950,461; 8,019,925; 9,146,237.

5. The average monthly rainfall or melted snowfall for a certain city is given in the table below. Make a line graph representing this table letting 2 cm. denote 1 inch. Follow the directions given on page 40.

Months	*Jan.*	*Feb.*	*Mar.*	*Apr.*	*May*	*June*	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*
Inches	2.8	2.30	2.56	2.70	3.59	3.79	3.61	2.83	2.91	2.63	2.66	2.71

6. The hourly temperatures on a certain day were as follows:

Time of day	*6:00 a.m.*	*7:00*	*8:00*	*9:00*	*10:00*	*11:00*	*12:00*	*1:00*	*2:00*	*3:00*	*4:00*	*5:00*
Temperatures	48°	49°	53°	55°	61°	65°	70°	72°	72°	70°	65°	63°

Make a line graph of this table and tell what the graph shows. Find the average temperature.

7. Make a bar graph of the number of points made by pupils in a written test recorded below.

No. of points	*12*	*11*	*9*	*8*	*7 or less*
No. of pupils	3	7	9	4	2

8. The average length of day from sunrise to sunset varies (changes) with the latitude. Represent by a line graph the following average lengths of days in latitude 45°, letting 1 cm. represent one hour.

Months	*Jan.*	*Feb.*	*Mar.*	*Apr.*	*May*	*June*	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*
Hours	9.1	10.4	11.9	13.5	14.9	15.6	15.3	14.1	12.6	11.1	9.6	8.8

9. Represent by a bar graph the fire losses in the city of Chicago given in the following table:

Years	*1912*	*1913*	*1914*	*1915*	*1916*	*1917*	*1918*
Number of millions of dollars lost	6.65	5.01	6.01	4.9	5.49	4.18	4.08

10. The table below gives the death rates caused by influenza and pneumonia in some of the large cities during the epidemic in 1918. Compute the increase in the death rate from 1917 to 1918. Make a bar graph of the increases and tell what the graph shows.

Death Rates Caused by Influenza and Pneumonia

Cities	*1917*	*1918*	*Increase*
Cleveland	13.9	16.0	2.1
Chicago	14.0	17.1	....
New York	15.2	18.8	....
Dayton	15.9	19.6	....
Los Angeles	12.5	16.4	....
Cincinnati	15.5	20.6	....
Louisville	16.3	21.0	....
Richmond	18.5	23.6	....
San Francisco	15.0	20.5	....
Boston	15.4	22.0	....
New Orleans	19.9	25.9	....
Philadelphia	17.1	24.2	....
Pittsburgh	18.2	25.4	....

11. Select the most convenient unit and then make a bar graph representing the facts in the table below, which gives the production of corn in several states.

Suggestion: Use only the first three figures for making the graph.

Production of Corn in Several States

States	*Number of Bushels*	*States*	*Number of Bushels*
Illinois	444,843,000	Indiana	208,522,000
Iowa	411,656,000	Ohio	162,859,000
Missouri	263,463,000	Kentucky	126,859,000

Compare by means of your graph, and by means of ratio, the productions of the following states: Iowa with Missouri; Iowa with Indiana; Missouri with Kentucky; each state with Illinois.

12. Make graphs representing the heights of the following buildings in New York City: Woolworth 750 feet, Metropolitan 700 feet, Singer 612 feet, Equitable 486 feet, Times 420 feet, Flatiron 286 feet. Represent by a bar the tallest building in your city. Find the ratio of each of the buildings above to the height of your tallest building.

13. The height of the average man is about 5 ft. 5 inches. The average weight for men 5 ft. tall is given in the following table for different ages. Make the graph.

Ages	*15–24*	*25–29*	*30–34*	*35–39*	*40–44*	*45–49*	*50–54*	*55–59*	*60–64*	*65–69*
Pounds	134	138	141	143	146	147	149	149	148	147

14. Represent graphically the average weights of boys and girls given in the following table:

Ages	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
Weight of boys	45	49	54	59	65	70	76	84	95	107	111
Weight of girls	43	47	52	57	62	69	78	88	98	106	112

15. The table below gives a record of a typical day's demand for electricity in New York. Make a graph of this table and explain the following facts shown in the graph: very little demand at 4 A.M.; the rapid increase from 5 to 7 A.M., and from 7 to 9 A.M.; the great demand at 11 A.M.; the sudden drop at 12:30; the largest demand at 5 P.M.; the slow decrease from 7 to 9 as compared with that from 5 to 7 and from 9 to 12 P.M.

A.M.

Time of day	*12*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
1000 kilowatts	65	53	42	39	38	41	58	75	105	147	165	168	165

P.M.

Time of day	*12:30*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
1000 kilowatts	135	158	187	180	180	233	198	159	154	135	118	90	72

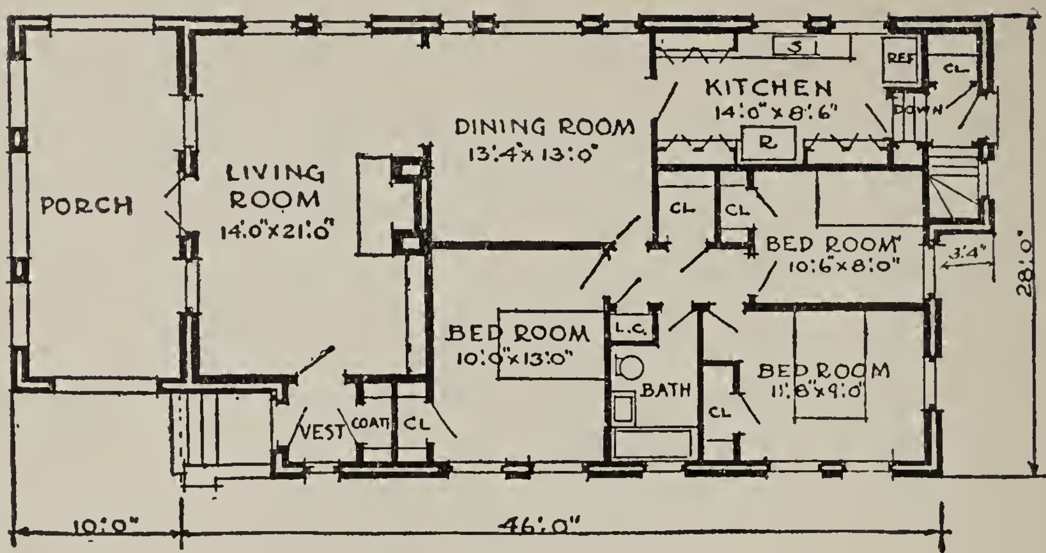


Fig. 58

16. Make a scale drawing of the floor plan (Fig. 58).

17. A department store reports the sales and profits for a period of five years as follows:

Year	*1918*	*1919*	*1920*	*1921*	*1922*
Sales in millions	4	$5\frac{1}{2}$	6.9	7.1	8
Profits in millions	.7	$1\frac{1}{2}$	1.6	1.6	2

Represent the table graphically.

18. On a normal business day the average number of passengers carried in and out of Chicago by the suburban trains of the Chicago and North Western Railway is approximately 60,000.

The number of trains necessary to handle this business is 166.

On August 1st, the first day of the street car strike, the suburban trains of this company carried 109,810 passengers.

The number of trains required for this business was 225.

On August 2nd the number of passengers carried was 134,704. The number of trains operated, 245.

On August 3rd the number of passengers carried was 135,619. The number of trains operated, 243.

On August 4th the number of passengers carried was 134,822. The number of trains operated, 242.

Make tabulated statements of these facts as follows:

	No. Passengers	*No. Trains*	*Passengers Per Train*
Normal	60,000	166	361
August 1st			
August 2nd			
August 3rd			
August 4th			

Represent by line graphs the passengers' increase and the train increase. Represent by a bar graph the number of passengers per train.

19. The following table shows the loss from absences in a high school for one year. Represent the facts in the table graphically.

Months	*Oct.*	*Nov.*	*Dec.*	*Jan.*	*Feb.*	*Mar.*	*Apr.*	*May*	*June 10 days*	*Year*
No. of different pupils absent	94	114	129	299	259	188	205	175	71	
Aggregate days absent	241	281	260	652	440	624	633	404	161	3,696
Per cent of daily absences	2.1	2.5	3.3	5.4	4.7	7.1	5.8	3.5	3.1	4.34

28. What every pupil should know and be able to do. Having made a study of this chapter the pupil should understand how numerical facts may be represented arithmetically as in tables, or geometrically as in graphs. He should be able to tell what a given bar graph or line graph shows, and he should know how to make a graph from a given table of numerical facts.

29. Typical exercises. One who understands this second chapter should be able to work the following problems:

1. Draw a bar graph representing these numbers: 2.50, 3.41, 4.23, 1.68.

2. Some of the longest rivers we read about in the study of geography of the United States expressed in thousands of miles are approximately of the following lengths.

Missouri-Mississippi 4.2, Yukon 2.2, St. Lawrence 2.1, Arkansas 2, Rio Grande 1.8, Columbia 1.4, Colorado 1.4. Make a bar graph representing these approximate lengths.

3. The temperature record for a certain day reads as follows:

Hour	*6 a.m.*	*7*	*8*	*9*	*10*	*11*	*12*	*1 p.m.*	*2*	*3*	*4*	*5*	*6*
Temperature	16°	18°	19°	19°	21°	22°	24°	24°	20°	18°	18°	16°	12°

Make a line graph showing the changes in temperature on that day.

4. Write a paper on the meaning and uses of graphs.

CHAPTER III

REPRESENTING NUMERICAL FACTS BY FORMULAS

How to Make a Formula

30. A third method of representing numerical facts. In Chapters I and II we have studied two methods of representing numerical facts, the *arithmetical* by which facts were arranged in the form of tables, and the *geometrical* by which they were represented in geometric figures, *e.g.*, in graphs. There is a third method, which we shall study in this chapter and which is sometimes the most convenient of the three. It represents numbers by means of letters. It is used not only in mathematics but in science, shop work, and engineering. The following exercises introduce this new method:

Exercises

1. On a sheet of centimeter squared paper lay the edge of a ruler, which is divided in inches, along one of the heavy lines. Using the centimeter as unit, measure carefully segments equal to 1 in., 2 in., ..., 8 inches and tabulate the results as follows:

Number of Inches..............	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Number of centimeters........	2.54	5.08						
Ratios........................								

Find the ratio of each number in the second row to the corresponding number in the first row. If this is done accurately, you

will see that the number of centimeters in a segment is approximately 2.54 times the number of inches.

The equation $c=2.54i$, stated in words, means *the number of centimeters* is 2.54 times *the number of inches.*

2. The price of oranges of a certain size is 30c a dozen.

a. Complete the following table which gives the price of oranges from 1 to 10 dozen.

Number of Dozen	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Price in cents.	30	60	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . . .

b. Make a line graph illustrating the facts given in the table above. To represent the price, let one side of a large square on the graphing paper represent 30.

c. From the graph determine the price of $4\frac{1}{2}$ doz.; $7\frac{1}{2}$ doz.; $8\frac{1}{2}$ doz.

d. From the graph determine how many oranges can be bought for \$0.60, \$1.80, \$2.10, \$2.70.

e. In the table above compare by means of ratios the numbers in the second row with the corresponding numbers in the first row. What relation do you find between price and the number of dozens bought?

f. Express the results of (*e*) in one general statement, giving the price in terms of the number of dozens.

g. Let the number n represent the *number of dozens*, and the number p the *price* in cents. Show that the statement in (*f*) may be expressed briefly in the form $p=n\times30$, or $p=30\times n$.

h. The statement $p=30\times n$ in words means: the number of cents is equal to 30 times the number of dozens.

i. When $n=1$, show that $p=30\times1=30$.

j. Let $n=2$, and show that $p=60$.

k. Show how to obtain all of the facts stated in the table and in the graph from the brief statement $p=30\times n$.

31. What is meant by the value of a literal number. In statements like $p=30\times n$, the literal number n may stand for *any* number, as, 1, 2, 3, etc. The

literal number p then stands for the corresponding numbers 30, 60, 90, etc. A number for which a literal number stands is a *value* of the literal number.

32. The meaning of the word formula. The statement $p=30\times n$ expresses the equality of the two numbers p and $30\times n$, and is therefore an *equation.*

Exercise 2, k (§30), shows that the equation $p=30\times n$ enables us to determine the price of any given number of dozens of oranges selling at 30 cents a dozen.

When an equation is used to state briefly a rule for obtaining numerical facts from other related facts it is a *formula.*

Thus, $p=30\times n$ is a *formula.*

The following exercises teach how to "make formulas":

Exercises

1. *a.* Let n represent the number of articles purchased, let p be the price of each, in cents, and let c be the total cost in cents. In the table to the right find the values of c for given values of n and p. Thus, when $n=1$, $p=10$, $c=1\times 10=10$, etc.

b. Make a formula for finding the cost c for n articles each sold at p cents.

c. Show that all the facts found in Exercise 1, *a*, can be obtained from the formula $c=n\times p$.

n	p	c
1	10	$1\times 10=10$
2	8	
3	16	
4	4	
5	18	
10	p	
n	6	
n	p	

2. *a.* Make a table giving the number of inches, i, corresponding to the number of feet, f, letting f take the values 1, 2, 3, ...

b. Make a statement expressing in words, the number of inches in terms of the number of feet. Translating this statement into symbols, make a formula for finding the values of i, Exercise 2, *a*, corresponding to values of f.

c. Using the facts given in the table of Exercise 2, *a*, make a graph of the formula $i = 12 \times f$.

3. *a.* A cubic foot of water weighs 62.5 pounds. Show that the number of pounds (weight) w, of a volume of v cubic feet is given by the formula $w = 62.5 \times v$. Translate this formula into words.

b. Tabulate corresponding values of v and w, Exercise (*a*) and make a graph of the equation $w = 62.5 \times v$.

4. *a.* A train traveling over equal distances in equal time-spaces is said to have *uniform motion.* The distance traveled in the unit of time is the *velocity* (speed, or rate) of the train.

Complete the table at the right, stating values of the distances, d, corresponding to values of the time, t, for a train having a velocity, r, of 20 miles an hour.

t	r	d
1	20	$1 \times 20 = 20$
2	20	
2.5	20	
$3\frac{1}{3}$	20	
4.75	20	
5	20	
6	20	
t	20	

b. In the table below find the values of d corresponding to given values of r and t.

c. Write a formula from which to find the distance, d, of a train traveling at a rate, r, for t hours.

t	r	d
1	30	$1 \times 30 = 30$
6	15	
12	40	
8	20	
15	10	
t	16	
18	r	
t	r	

5. Make a formula for finding the number of feet, f, from a number of yards, y. Translate this formula into words.

6. A boy earns d dollars a week for n weeks. State a formula for finding his total earnings t.

7. If n oranges cost C cents, state a formula for finding the cost, c, of one orange.

8. Make a formula for finding the number of gallons n, of oil, which pass through a pipe in m minutes, at the rate of 2 gallons a minute.

9. A river flows at the rate of r mi. an hour. Find the distance, d, an object will float in t hours.

10. Make a formula for finding the quotient, q, when the dividend is D and the divisor d.

11. A boy is n years old. What was his age, a, 5 years ago? What will be his age, A, 5 years from now?

12. A girl saves $2 a week for n weeks. Make a formula for finding her total savings x.

13. A girl receives 10 cents each time she does the shopping for her mother. Make a formula for finding the amount, t, which she earns in n days, if she goes shopping twice a day.

33. Various uses of the formula. As we go on in the study of mathematics we shall meet formulas frequently because the formula saves time and effort in the solution of many problems. Formulas are used in other school subjects. Thus, in science the formula helps us to find the distance a particle falls in a given time, and in physics many laws are stated as formulas. The formula plays an important part in every-day life. The automobile owner determines the horsepower of his gasoline engine by means of a formula, the business man may use it to compute interest on a sum of money, and the machinist to find the length of belting connecting two pulleys. Scientists, engineers, machinists, surveyors, insurance men, and many others use formulas, and should know the correct way in which to work with them. The importance of the formula is one of many reasons for studying mathematics.

Perimeter Formulas

34. Polygons. On notebook paper mark points A, B, C, D, E, and F, as shown in the diagram (Fig. 59) and join them with line segments. The figure thus formed is called a **polygon**. The word *polygon* comes from the Greek and means a figure with *many angles.*

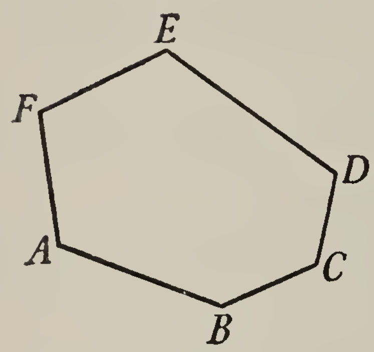


Fig. 59

What is the least number of segments needed to form a polygon?

The points A, B, C, etc., are the *vertices*, the segments AB, BC, CD, etc., are the *sides* of the polygon.

Measure the sides and find the sum

$$AB+BC+CD+DE+EF+FA.$$

The sum of the sides of a polygon is the *distance around*, or the *perimeter* of the polygon.

A polygon is called a *triangle, quadrilateral, pentagon, hexagon,* . . . . according as it has 3, 4, 5, 6, ... sides.

A polygon is *equilateral* when all the sides are of *equal length.*

Exercises

1. Measure each side of triangle ABC (Fig. 60) and find the perimeter by adding the lengths.

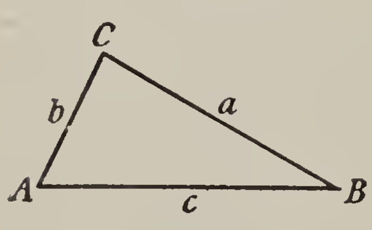


Fig. 60

Denoting the perimeter by p, state the result in the form of an equation.

When the sides are not measured, the perimeter, p, of triangle ABC is given by the formula $p=a+b+c$.

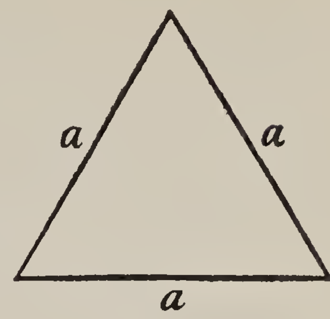


FIG. 61

2. Without measuring the sides find the perimeter of the equilateral triangle (Fig. 61) and state the result in the form of a formula.

3. What is the perimeter, p, of a triangle whose sides are $3x$ ft., $4x$ ft., and $5x$ feet? State the result in the form of an equation.

4. For each of the following equations sketch at least one polygon whose perimeter is expressed by the equation:

1. $p=3x$. 2. $p=5x$. 3. $p=6x$. 4. $p=8x$.

Suggestion: Change $3x$ to $x+x+x$, $6x$ to $3x+2x+x$ or $4x+x+x$, etc.

5. Find the perimeter of each of the polygons in Exercise 4 when $x=2.75$ cm.

6. Write the equations in Exercise 4 when the perimeter $p=148$.

7. The perimeter of an equilateral hexagon is 120 inches. Find the length of a side.

Solution: Let x be the number of inches contained in the side.

Show that $120=6x$.
Show that $x=\frac{1}{6}$ of $120=\frac{1}{6}\times 120$.
Hence, $x=20$.

8. The perimeter of an equilateral quadrilateral is 24. Find a side by means of an equation, as shown in Exercise 7.

9. The perimeter of an equilateral pentagon is 60. Find each side.

10. The perimeter of an equilateral decagon (10-sided polygon) is 28. Find each side.

35. Symbol for hence and therefore. The symbol used to denote *hence* or *therefore* is $\therefore$.

Addition and Subtraction

36. A third way of finding the perimeter. We have seen that the perimeter may be found in two ways.

1. *Arithmetically.* Each side is measured and the measures are added. The resulting arithmetical sum is the perimeter.

2. *Algebraically.* This means that the sides are added without finding the length of each. Thus, $p=a+b+c$. The algebraic sum $a+b+c$ denotes the perimeter. The equation $p=a+b+c$ is a formula for finding the value of the perimeter when the values of a, b, and c are given.

The following is a *third* way of finding the perimeter.

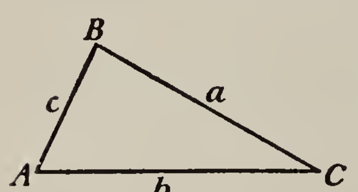


Fig. 62

Draw a line of indefinite length, as OX (Fig. 63).

Open the compass a distance a (Fig. 62) and from O lay off on OX the segment OD equal to a.

Similarly, from D lay off a distance DE equal to b in the direction of X, and from E lay off EF equal to c.

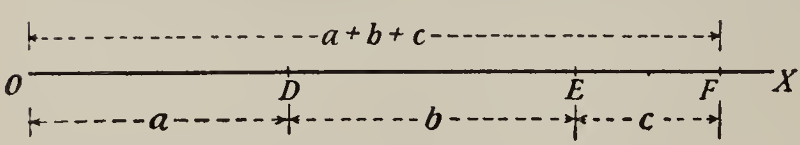


Fig. 63

OF is the perimeter of triangle ABC.

The line segment OF may now be measured to determine the arithmetical value of $a+b+c$.

Exercises

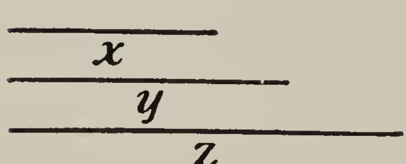


Fig. 64

1. Draw three segments, x, y, and z (Fig. 64) and find the sum in the three ways shown above.

To find the line segment representing the sum proceed as in Fig. 63, and then mark the diagram as shown there.

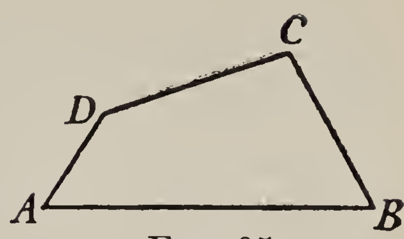


FIG. 65

2. Find in three ways the perimeter of the quadrilateral $ABCD$ (Fig. 65).

3. Draw two unequal segments a and b. Construct the sums $a+b$ and $b+a$. Measure each sum and show that $a+b=b+a$.

37. The law of order in addition. Exercise 3 illustrates the principle that *the value of a sum remains unchanged when the order of the addends is changed.* This is called the *law of order in addition.* This law helps us to simplify the adding of numbers by arranging them first in an advantageous order. For example, the terms in the sum $475+210+25$ may be taken in the order

$$475+25+210=500+210=710$$

To indicate that we wish to add first 475 and 25 and then 210, the last statement may be written in the following forms:

$$(475+25)+210=500+210=710,$$
or $$[475+25]+210=500+210=710,$$
or $$\{475+25\}+210=500+210=710.$$

38. Parentheses. The symbols ( ), [ ], { } have been used to show the *order* in which numbers are to be added, subtracted, or multiplied. Thus, $4\times(2\times3)$ means that first 2 is to be multiplied by 3 and the product is then multiplied by 4.

EXERCISES

In each of the following exercises state the meaning of the symbols and then add, subtract, and multiply in the order indicated.

1. $(324+15)+75$.
2. $(24+8)-22$.
3. $(15-2)+4$.
4. $5+\{7-3\}$.
5. $16-[7+3]$.
6. $18-[10-6+2]$.

7. Rearrange in the most advantageous way and then add: $240+325+60$; $736+298+64$; $350+287+50$.

8. State the meaning of each of the following expressions:
$(6+7)+3$; $26+(65+125)$;
$(a+b)+(c+d)$; $(10+15)+50+13$.

The symbols ( ), [ ], and { } are called **parentheses, brackets,** and **braces** respectively.

39. Finding the difference between two segments.

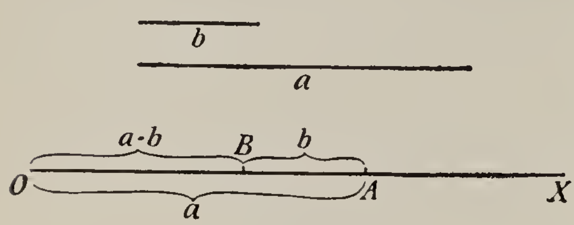


Fig. 66

Draw two segments a and b (Fig. 66), making a greater than b.

Draw an indefinite line, OX.

On OX lay off $OA=a$.

From A lay off, in the direction AO, the segment $AB=b$.

The segment OB is the *difference* between a and b. In the form of an equation this may be written

$$OB=a-b$$

Exercise

Draw three segments a, b, c (Fig. 67) making $a>b>c$. Then construct the following sums and differences, marking all segments as shown in Figs. 63 and 66:

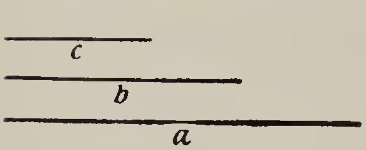


Fig. 67

$a+b-c$; $a+c+b$; $a+c-b$; $b-c+a$; $a-c+b$.

40. Monomials. Terms. Numbers like $30\times n$, $20\times t$, $r\times t$, are *monomials*, or *terms*. The numbers 30 and n are *factors* of $30\times n$, 20 and t are *factors* of $20\times t$, r and t are *factors* of $r\times t$.

41. Similar terms. Monomials having a *common* (the same) literal factor, as $4\times a$ and $8\times a$, are **similar terms.**

42. Coefficient. In the monomials $2\times n$, $30\times x$, $20\times t$, the numerical factors 2, 30, 20, are called **coefficients** of the literal factors n, x, and t respectively. It is customary to write products like $2\times n$, $30\times w$, $20\times t$, briefly $2n$, $30w$, $20t$, omitting the multiplication sign. It must be remembered that such numbers have two meanings. For example, $3m$ means $m+m+m$, and 3 multiplied by m. State two meanings of each of the products above.

When no coefficient is stated, as in the numbers a, x, y, the coefficient is *understood* to be 1. Thus a means $1a$, x means $1x$.

Exercises

1. State the following products in a brief form: $6\times s$, $5\times m$, $80\times d$, $p\times r$.

2. Give several examples of monomials; of similar monomials; of dissimilar monomials.

3. State two meanings for each of the following: $5y$, $4x$, $7a$. What is the meaning of ab, xy, rt?

4. Find the values of $15t$ when $t=1$, 2, 6, 7.5, 10.75, $12\frac{2}{3}$.

43. Polynomials. Numbers like $a+b+c$ and $x+y+z$, which consist of two or more *monomials* (terms), are called **polynomials** (*having many terms*). A polynomial with only *two* terms, as $a+b$, is a **binomial.** A polynomial containing *three* terms, as $a+b+c$, is a **trinomial.**

44. Adding and subtracting similar terms. We have seen that

$$2a \text{ means } a+a,$$

and that $\quad 3a$ means $a+a+a,$

Hence, $2a+3a$ means $(a+a)+(a+a+a),$

or $a+a+a+a+a$, or $5a$.

It follows that

$$2a+3a=5a.$$

This shows that the similar terms $2a$ and $3a$ may be *combined*, or collected, *by adding the coefficients* 2 and 3, *and then multiplying the sum by the common factor* **a**.

Numbers not having a common factor cannot be combined. Thus in the sum of a and b the addition must remain *indicated*, as $a+b$.

Exercises

1. By combining similar terms, reduce each of the following polynomials to a simpler form. Arrange the work as follows:

$$4m-\tfrac{1}{3}m=(4-\tfrac{1}{3})m=\tfrac{11}{3}m.$$

$8x+4x$; $10a+3a+a$; $\frac{1}{2}x+\frac{3}{4}x+\frac{1}{8}x$; $5.2m+6.7m-3.1m$.

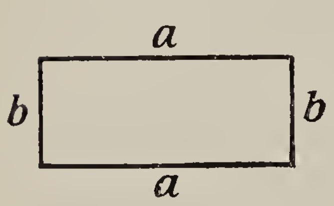


Fig. 68

2. The lengths of the sides of the rectangle (Fig. 68) are respectively a, b, a, and b. Write the perimeter in the simplest form.

3. Find the perimeter of each of the figures below (Fig. 69) and write the results in the simplest form. Find the value of each of the resulting polynomials for $a=6$, $b=3$, $c=2$, $d=4$, $f=1$.

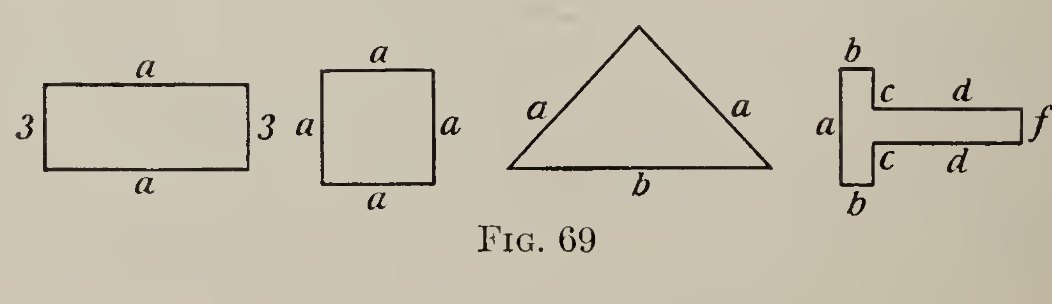


Fig. 69

4. The number of pupils in a class is n. Five pupils are withdrawn. Express as a binomial the number left.

5. A man had a acres of land and sold s acres. How many had he left?

6. The difference of two numbers is 20. The smaller number is n. What is the larger?

7. Express in symbols: $\frac{1}{6}$ of a number n diminished by 4.

8. Five times a number x is increased by 3, and the sum is divided by 7. Write the result in symbols.

Write the following polynomials in the simplest form.

9. $2x+7x-4x+6x$.

Solution: $2x+7x-4x+6x=2x+7x+6x-4x=15x-4x=11x$.

10. $3a+7a+2a-a$.

11. $\frac{1}{3}m+\frac{5}{6}m+\frac{3}{5}m-\frac{1}{2}m$.

12. $2.3t+1.8t-.9t+6.1t$.

13. $20m+6m+17n-3n$.

Solution: $(20m+6m)+(17n-3n)=26m+14n$.

14. $14x-3.6x+8.4y-1.9y$.

15. $1\frac{2}{3}a-\frac{1}{4}a+\frac{3}{5}b-\frac{1}{2}b$.

16. $12.6c-.25c+18.3c$.

17. $(8x+4x)+(10x-2x)+3$.

18. $\frac{1}{3}a+\frac{3}{8}a+\frac{1}{4}a+5$.

Find the value of each of the following for $a=6$, $b=3$, $c=2$.

19. $a+2b+3c$.

Solution: $a+2b+3c=6+(2\times3)+(3\times2)=6+6+6=18$.

20. $2a+\frac{b}{2}-2c+4$.

21. $\frac{a-c+2b}{3+2a}$.

22. $\frac{2c+1.2a-.6b}{a+b+c}$.

23. $\frac{3c+2b-a+1}{8c-b-a}$.

24. $\frac{\frac{1}{2}a-b+c+6}{a-\frac{2}{3}b+\frac{3}{4}c}$.

25. $\frac{6.2a-2.4b+c}{\frac{3}{4}a+1.2b+.6c}$.

Equations

45. How to solve equations. We have seen (§§30–32) that numerical facts may be represented by equations. For example, the equation $c=30n$ states that the number of cents paid for oranges is 30 times the number of oranges. In studying perimeters (§34) we were able to find the lengths of sides of polygons by means of equations, such as $6x=120$. In the study of mathematics the equation is a powerful instrument in solving problems and in representing numerical facts in a brief form. Hence, we must learn to work with equations.

An equation, as $6x=30$, states the equality of the numbers $6x$ and 30. The number $6x$ to the *left* of the equality sign is called the *left side*, or the *left member*, of the equation; the number to the *right* is the *right side*, or *right member*. The literal number, x, is called the *unknown number*. The process of finding a value of the unknown number for which both members are the same is called *solving the equation.*

Exercises

Solve the following equations:

1. $2x=19$.

Solution: $2x=19$.

Dividing each member by 2, we have

$$\frac{2x}{\cancel{2}}=\frac{19}{2}$$

$$\therefore x=9.5.$$

2. $3x=18$.
3. $2x=11$.
4. $5x=20$.
5. $7x=13$.
6. $2.5x=50$.
7. $3.75x=100$.

For each of the following problems first state the equation and then solve it. Arrange your work as in the solution of Exercise 8, below.

8. A certain number multiplied by 6 gives the product 42. Find the number.

Solution: Let n be the required number.

Then the problem is briefly stated in the form of the equation $6n=42$.

To find n, divide each member of the equation by 6.

This gives $\frac{\cancel{6}n}{\cancel{6}}=\frac{42}{6}$

$\therefore n=7.$

9. A train traveling at the rate of 35 miles an hour made a distance of 75 miles. How much time did it take?

10. A field containing $\frac{5}{8}$ of an acre is sold for \$300. What is the price per acre?

11. In a given time the minute hand of a clock passes over 12 times as many minute spaces as the hour hand. Over how many minute spaces does the hour hand pass in 50 minutes?

12. How long will it take a man to ride 60 mi. at the rate of 18 mi. an hour?

13. A man earns \$125 in 16 days. How much does he get per day?

46. Solving equations by using an axiom. In the solution of equations (§45) it has been taken for granted that when both members of an equation are divided by the same number, the quotients are also equal.

Thus, if $2x=10,$

it is assumed that $\frac{2x}{2}=\frac{10}{2},$

or that $x=5.$

This assumption may be stated in the form of a general principle as follows: *If equal numbers are divided by the same or equal numbers, the quotients are equal.*

Such statements when assumed to be true are called *axioms.*

The equation above is solved by dividing both members by 2. The principle used in the solution of the equation is called the *division axiom.* This axiom is to be used in solving each of the equations below.

Exercises

Solve the following equations explaining each step, and arrange the work as shown in Exercise 1, below:

Solution:	*Authorities:*
1. $2a=16$	
$\frac{\cancel{2}a}{\cancel{2}}=\frac{16}{2}$	If equal numbers are divided by the same number the quotients are equal.
$a=8.$	By reducing fractions.

2. $11a=22.$
3. $19m=76.$
4. $3.5x=24.5.$
5. $11h=17.$
6. $93n=60.$
7. $45x=120.$

In the following equations find the values of the unknown numbers approximately to the nearest third figure:

8. $1.23y=532.$
9. $287x=5.89.$
10. $21.2n=62.1.$
11. $7.5k=28.2.$
12. $.57r=24.2.$
13. $1.32t=226.$
14. $118s=237.$
15. $.231x=462.$

Solve the following problems by means of equations, arranging the work as shown in Exercise 16, below:

16. John is able to solve twice as many problems as James. Mary can solve three times as many as James. Together they solve 48 problems. How many problems does each solve?

Solution: Let x be the smallest number, *i.e.*, the number of problems James solves

Then $2x$ is the number John solves

and $3x$ is the number Mary solves

$\therefore x+2x+3x=48$ since together they solve 48 problems

$6x=48$, by combining similar terms

$\therefore x=8$, the number James solves

$2x=16$, the number John solves

$3x=24$, the number Mary solves.

Notice that the solution of the problem involves the following steps:

a. The problem is read carefully, to find what number the problem calls for. In this case there are three unknown numbers.

b. One of the unknown numbers is denoted by a letter, as "x." In this case x denotes the number of problems solved by James.

c. The other unknown numbers are now expressed in terms of "x." Thus, John and Mary solve $2x$ and $3x$, respectively.

d. Then the equation is formed and solved.

These suggestions, if followed, will be helpful in solving the problems below:

17. A sum of $32 is to be broken up into two parts, one part being seven times as large as the other. Find the two parts.

18. Sixty rods of fence are available to inclose a rectangular field. The field is to be five times as long as it is wide. Make a sketch of the field and find the dimensions.

19. A man is three times as old as his son. The sum of their ages is 48 years. Find the age of each.

20. A class is to be formed having twice as many boys as girls. If there are to be 30 pupils in the class, find the number of girls.

21. The sum of three numbers is 80. The second is twice as large as the first and the third is five times as large as the first. Find the three numbers.

Solve the following equations:

22. $x+4x=200.$

23. $5x-x=60.$

24. $7x+x=30.$

25. $8x-x=35.$

26. $5x+3x=40.$

27. $5x+4x-x=70.$

47. What every pupil should know and be able to do. In the unit just completed the meaning of *formula* and *equation* has been developed. The pupil is now expected:

1. To use correctly the expressions: polynomial, monomial, binomial, trinomial, term, similar terms, value of a literal number, coefficient, formula, equation, polygon, triangle, quadrilateral, pentagon, hexagon, vertex, side, perimeter.

2. To understand the arithmetical, geometrical, and algebraic forms of representing numerical facts, *i.e.*, the representation by table, graph, and formula.

3. To know the meanings and names of the symbols ( ), [ ], { }.

4. To find the value of expressions of the form $6x$; $a+b+c$; $2x+3x+x$; $\frac{a+b+c}{4-2a}$; where the values of the literal numbers are whole numbers, common fractions, or decimal fractions.

5. To solve equations of the form $5x=12$; $2x+7x=4$.

6. To solve simple problems leading to equations of the forms given in 5.

7. To know the following laws:

a. Law of order in addition.

b. Division axiom.

48. Typical problems and exercises. Pupils should be able to work problems and exercises of the type given below:

1. The price of oranges of a certain size is 50 cents a dozen. Find the price of 2, 3, 4, ..., 10 dozen.

Represent these facts in tabular form; in graphical form; in a formula.

Show how to obtain the facts in the table from the formula.

From the graph determine the price of $4\frac{1}{2}$ doz., $6\frac{1}{2}$doz., 8 doz.

2. Translate into words the statement $i=12f$.

3. The perimeter of an equilateral pentagon is 35 inches. By means of an equation find the length of each side.

4. John solved twice as many problems as Mary, and James solved three times as many as Mary. Together they solved 36 problems. How many did each solve?

5. Draw a triangle and find the perimeter by measuring each side and then adding the results; by drawing the sum and then measuring the sum.

Find the value of each of the following, if $a=6$, $b=\frac{1}{2}$, $c=1.2$.

6. $\frac{1}{3}a+\frac{3}{8}a+\frac{1}{4}a$.

7. $(8a+4b)+(6a-2c)$.

8. $8.5c+1.16a-3b$.

9. $\dfrac{3.2a-4.1b+2c}{\frac{1}{2}a-3b}$.

Solve the following equations:

10. $45x = 120$.

11. $2.12n = 62.1$.

12. $5x + 4x - x = 70$.

13. State the following laws:
The division axiom.
The law of order in addition.

14. Write a paper on one of the following topics:
a. The value of the formula.
b. The equation as a tool for solving problems.

CHAPTER IV

A STUDY OF ANGLES

How Angles Are Used

49. Meaning of angle. After studying the line segment, which is the simplest geometric figure, we are now able to extend our study of geometry to more complicated figures made up of several line segments. A figure formed by two lines (Fig. 70) is called an *angle*. The word comes from the Latin *angulus*, meaning corner.

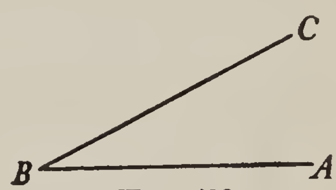


Fig. 70

More precisely, we may say that an angle is a figure formed by two straight lines, as BA and BC, meeting at the same point (Fig. 70). The straight lines are the *sides*, arms, or legs of the angle; the point, B, where the sides meet, is the *vertex*.

Exercises

1. Angles are found all about us. Point out some angles in the classroom, and the sides and vertex of each.
2. Point out several angles outside of the classroom.
3. Draw an angle on the blackboard.
4. Make an angle by folding a sheet of paper.
5. Draw a polygon and point out the angles in it.

50. Uses of angles. A knowledge of angles is important. Builders and architects use angles in plan-

ning and constructing our homes. We shall learn how the surveyor finds unknown distances which he cannot measure directly, such as the height of a tree (Fig. 71), the height of a tower or mountain, or the width of a river (Fig. 72). It will be shown how the astronomer, by a knowledge of angles, determines the position of the stars and planets, and determines the exact time, and how a knowledge of angles enables the navigator guiding his ship along the coast, to avoid hidden and dangerous rocks (Fig. 103). Engineers, designers, artists, and many others, make use of angles in doing their work.

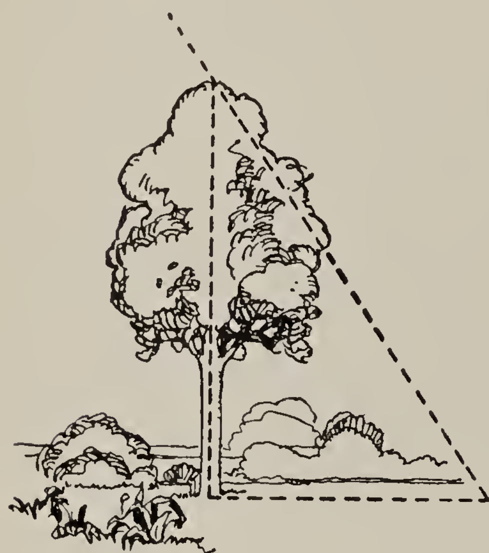


Fig. 71

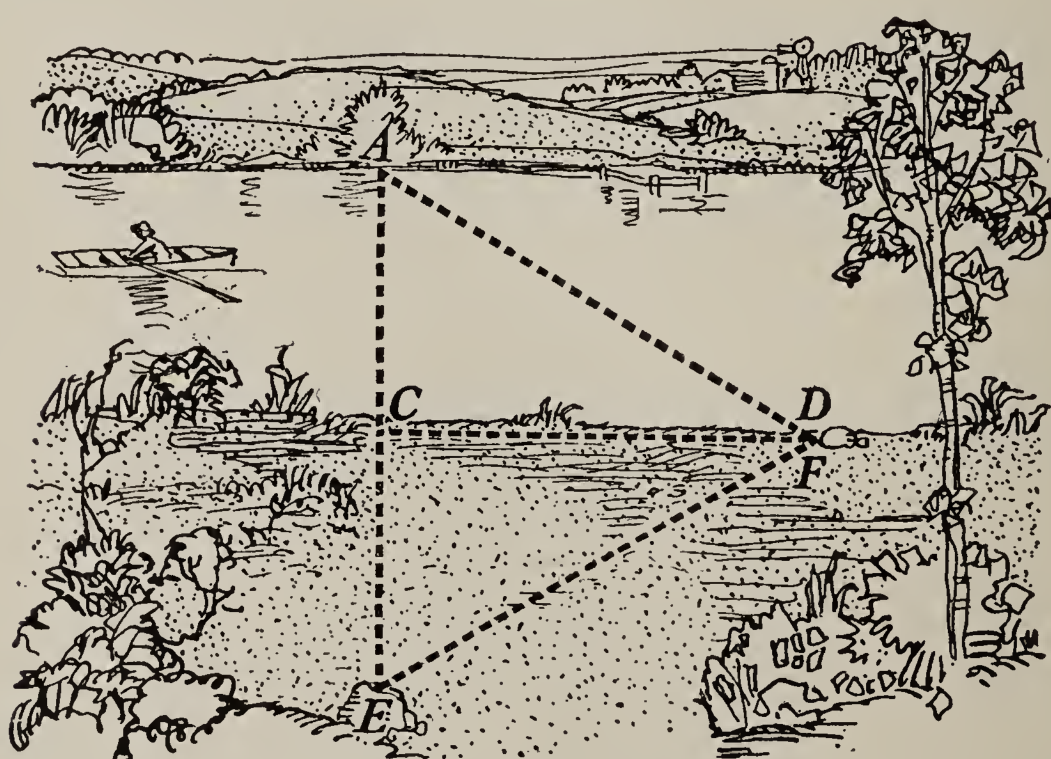


Fig. 72

By carefully studying this chapter we shall gain a thorough understanding of angles. We shall measure and draw angles to learn about their size. We shall learn the names of various kinds of angles. We shall study the relations between angles in the same geometric figure, as in the triangle or in several lines intersecting in the same point (Fig. 73), and use these relations to solve some problems in angles.

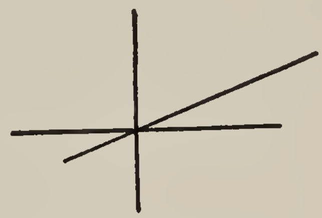


Fig. 73

We have said that an angle is a figure formed by two lines. It should not be inferred that two lines always form an angle. Point out lines in the class room which do not form an angle, no matter how far the lines may be extended. Such lines are known to us as *parallel lines*. The last part of this chapter takes up the study of angles formed by two parallel lines intersected by a third line (Fig. 74).

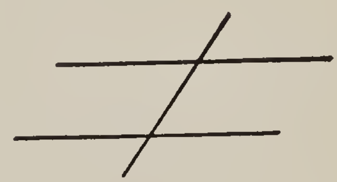
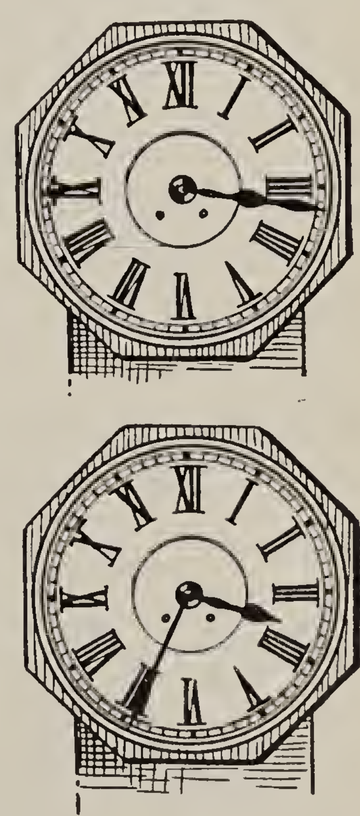


Fig. 74

51. Size of an angle. In the clock (Fig. 75) the hands are shown to be together. Since the minute hand of a clock turns faster than the hour hand, a turn of the minute hand separates the two hands, as in Fig. 76. The hands then *form an angle*. When the hands of a clock move, the angle changes in size.



Figs. 75, 76

An angle may be formed by keeping point B (Fig. 77) fixed and then

turning a line from the position BC in the *clockwise* direction until it takes the position BA; or by turning a line from BA in the *counterclockwise* direction until it takes the position BC. *The size of the angle depends entirely on the amount of turning necessary to carry the moving line from one side to the other, and not on the length of the sides.*

Fig. 77

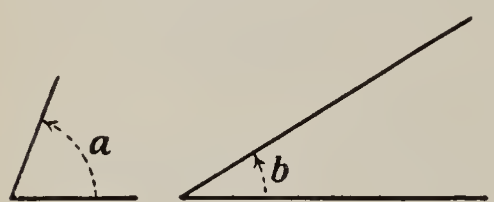


Fig. 78

Which of the two angles (Fig. 78) is the larger? Give a reason for your answer.

The curved arrow shown in the diagram is used to indicate the *direction* and *amount* of turning.

Two angles are *equal* if the same amount of rotation is needed to form them. If two angles are equal they can be made to *coincide* (fit).

Cut two angles from paper, and test them as to equality by placing one on the other. Tell which of the two is the larger angle.

52. Symbols used to denote angles. If we are to discuss angles, we must have a way of naming them. The symbol for the word *angle* is $\angle$. For *angles* it is ∠s.

There are various ways of naming an angle. We may use three letters (Fig. 79) one on each side and one at the vertex, and refer to the angle as $\angle ABC$, *using the vertex letter as the middle letter.*

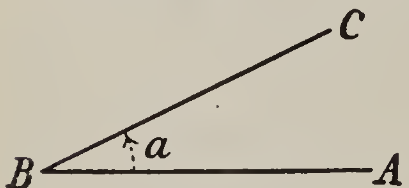


Fig. 79

A briefer notation is $\angle B$, which really means "an angle whose vertex is B."

Sometimes a small letter, as a, is written within the angle, and the angle is then called a.

EXERCISES

1. How many angles do you see in the triangle (Fig. 80)? Make a drawing of the triangle, and name each of the angles in the three ways described in §52. Thus, $\angle A$, a, $\angle BAC$, etc.

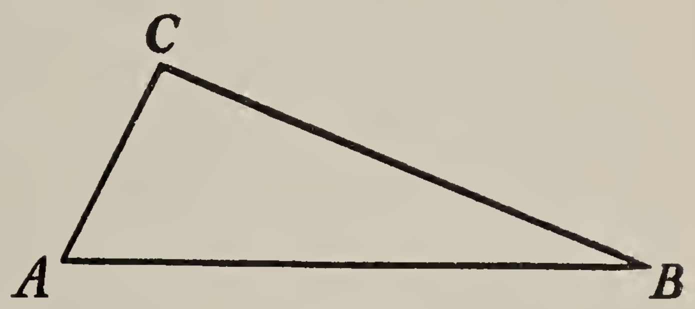


FIG. 80

2. How many angles are there in Fig. 81? Name each angle in three ways.

3. Draw three lines passing from the same point (Fig. 82). Name in three ways each of the three angles.

4. Draw a figure like Fig. 83 and name the angles formed.

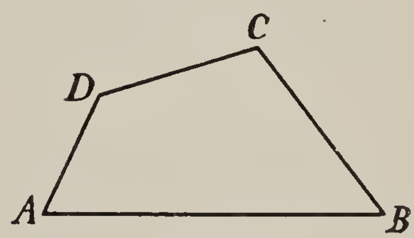


FIG. 81

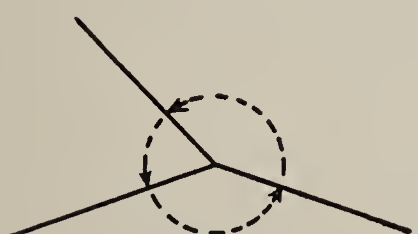


FIG. 82

FIG. 83

53. How angles are classified. Draw two lines in the position AB and AC (Figs. 84–88). Let a moving line turn about point A in the counterclockwise direction from the side AB to AC.

When the amount of rotation is equal to a *quarter turn* (Fig. 84) $\angle BAC$ is called a *right angle.*

An angle *less* than a right angle is *acute* (sharp) (Fig. 85).

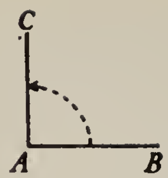


Fig. 84

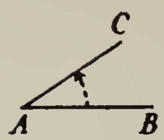


Fig. 85

Fig. 86

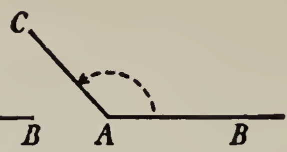


Fig. 87

Fig. 88

When AC makes a *half* turn (Fig. 86) a *straight* angle is formed. Thus, *the sides of a straight angle are in the same straight line on opposite sides of the vertex.*

Angles less than a straight angle and greater than a right angle are *obtuse* (blunt) (Fig. 87).

If the line AC makes a *complete* turn (Fig. 88) the angle is a *perigon* (round angle).

Exercises

1. State a time of the day when the hands of the clock form a right angle; a straight angle.

2. Point out obtuse angles in the classroom.

3. Open the arms of a blackboard compass so as to form an acute angle; a right angle; a straight angle.

4. Make a sketch of each of the following: right angle; acute angle; straight angle; obtuse angle; perigon.

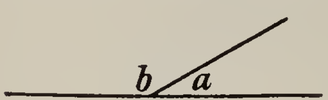


Fig. 89

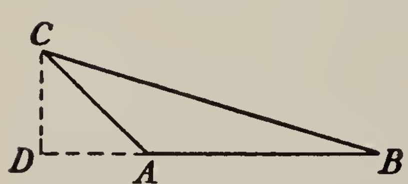


Fig. 90

5. Classify the angles a and b (Fig. 89).

6. Open your notebook, making two edges form a right angle; an obtuse angle; a straight angle.

7. Name an acute angle (Fig. 90); a straight angle; an obtuse angle; a right angle.

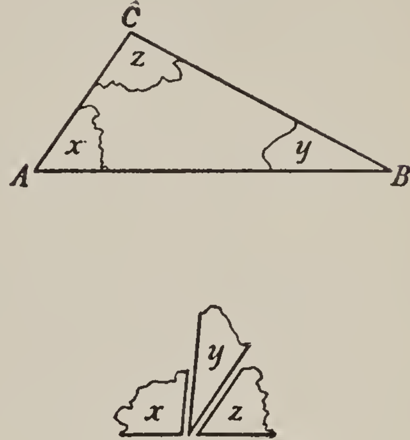
8. Classify the angles A, B, and C of triangle ABC (Fig. 90) and $\angle D$.

9. Name the largest angle (Fig. 90); the smallest angle.

10. Write in symbols: angle B is less than angle D; angle BAC is greater than angle B; angle DCA is equal to angle CAD.

11. How many right angles are there in a straight angle? In a perigon? How many straight angles are there in a perigon?

12. Draw a triangle ABC (Fig. 91) making the base about 8 cm. long. Mark the angles x, y, and z, respectively. Cut the triangle from the paper and tear off the corners as shown in the diagram. Place them adjacent to each other as in Fig. 92. Show that *the sum of the three angles of a triangle is a straight angle.* This is an important fact of geometry.



FIGS. 91, 92

13. Draw a quadrilateral. Tear off the corners, and show as in Exercise 12 that the sum of the four angles is a perigon.

MEASURING ANGLES WITH THE PROTRACTOR

54. Protractor. The *protractor* (Fig. 93) is an instrument used mainly for *measuring* angles. The curved rim is divided into 180 equal parts, every tenth of which is numbered. A line drawn from O, the mid-point of the straight edge, through a mark B on the rim, as OB, forms with the zero-lines, OX and OY, angles whose sizes may be read off on the outer and inner readings, respectively. Thus the measure of the straight angle XOY is 180, the measure of angle XOB is 30.

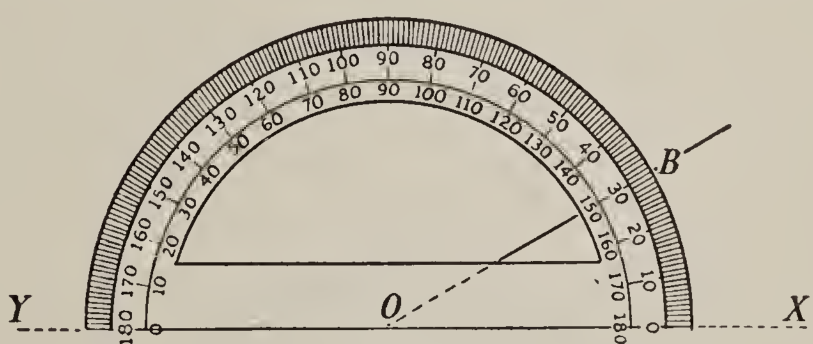


FIG. 93

If from the point O, lines are drawn to the marks on the rim, they divide the straight angle into 180 equal small angles, each of which is a *degree* (°).

The division of a straight angle into 180 equal parts is brought down to us from the Babylonian astronomers. For scientific purposes it would have been more convenient to use the metric system of dividing into 10, 100, 1000, etc., equal parts. Accordingly a right angle would be divided into 100 rather than into 90 equal angles. Only very recently attempts have been made in France to establish such a system.

The division of a round angle into 360 equal parts, or of a straight angle into 180 equal parts, originated as follows. The Babylonian astronomers believed that the sun revolved around the earth in 360 days. The daily part of a revolution was represented by the 360th part of a perigon. A knowledge of the triangle having equal sides probably led them to use one of the 3 equal angles as a unit. This unit, being contained 6 times in the perigon, was accordingly subdivided into 60 equal angles. This led to further division of these angles into 60 equal parts. This is known as the *sexagesimal system.* In this system every unit is divided into 60 equal parts. It is interesting to note that even today, modern civilization is still influenced by ancient Babylonian science in the reckoning of time and angles. For we divide an hour into 60 minutes, and a minute into 60 seconds. Similarly, the circle is divided into 360 equal parts called *arc degrees.* A degree of the arc is divided into sixty *minutes,* and a minute of arc into sixty *seconds.*

55. Units of angular measurement. We have seen that the 360th part of a *perigon* is a *degree;* that a degree is also the 180th part of a straight angle, and the 90th part of a right angle.

A *degree* may be divided into 60 equal parts. They are called *minutes* (′).

A *minute* may be divided into 60 equal parts, called *seconds* (″). These facts are summarized in the *table of angular measure* below. Since this table is used in future problems it should be memorized.

TABLE OF ANGULAR MEASURE

1 perigon	=	360°
1 st. ∠	=	180°
1 rt. ∠	=	90°
1°	=	60′
1′	=	60″

EXERCISES

1. Reduce 20° 15′ 18″ to seconds.

Solution:
$$\begin{aligned} 20° &= 1200' = 72000'' \\ 15' &= 900'' \\ 18'' &= 18'' \end{aligned}$$

Adding, $20°+15'+18''=72918''$

2. Reduce 17° 59′ 20″ to seconds.

3. Reduce 435′ to degrees and minutes.

4. Reduce 1026″ to degrees, minutes, and seconds.

5. Measure ∠*XOA* (Fig. 94).

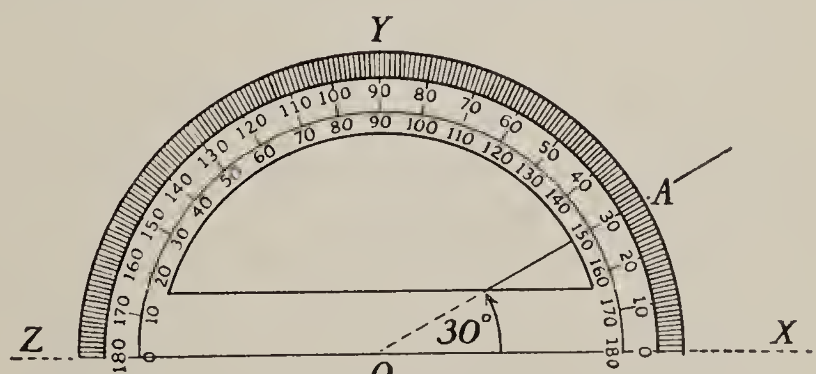


FIG. 94

Explanation: Place the protractor with the center on the

vertex O and make the line from the center to the zero-mark on the right of O fall along the side OX.

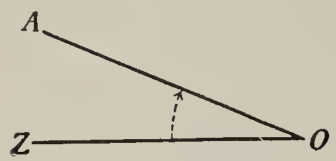


FIG. 95

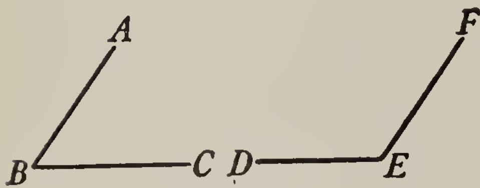


FIG. 96

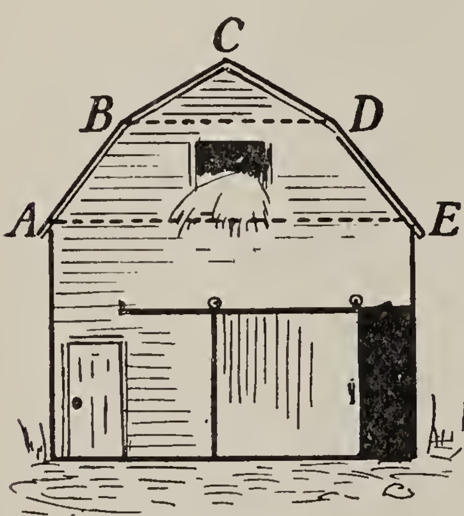


FIG. 97

From the zero-mark pass along the rim to the point where OA, or the extension of OA, intersects it.

Then take the reading at this point. This is the measure of $\angle XOA$.

Hence, $\angle XOA = 30°$.

6. Measure $\angle ZOA$ (Fig. 95).

Suggestion: Place the zero-line of the protractor along OZ. From the left-hand zero-mark pass along the rim to the point where OA, or the extension of OA, intersects it. Write the result near the vertex inside of the angle. To check your result measure the angle again by placing the zero-line along OA.

7. Measure $\measuredangle$ ABC and DEF (Fig. 96). Check your results, as in Exercise 6.

8. Fig. 97 is a picture of one end of a barn. Find by measuring what angles the rafters AB and BC form with each other. Find at what angle the rafter AB inclines toward the horizontal beam AE.

9. Draw two angles. Measure each, and find the ratio to two figures.

10. Draw a triangle whose longest side is 8 centimeters. As in §53 classify each angle of the triangle, and record the results in a table like that on page 85.

Estimate the number of degrees in each angle. Record the results in the table.

Measure each angle, and record the results.

By taking the difference between degrees estimated and degrees measured, compute errors, and record them in the column headed *errors*.

Find the sum of the number of degrees obtained by measuring.

Angles	*Classification of Angles (Acute, Obtuse, Etc.)*	*Number of Degrees*		*Errors*
		Estimated	*Measured*	
A				
B				
C				
Sums				

Compare your sum with the results of the other pupils of the class. What do you here find to be the number of degrees in the sum of the angles of a triangle?

56. Sum of the angles of a triangle. In Exercise 12 (§53) we have seen that the sum of the angles of a triangle is a straight angle. In §55, we have found by measurement that the sum of the angles of a triangle is 180 degrees. This may be expressed in symbols by means of the equation, $a+b+c=180$.

This equation is used by the surveyor as a formula for finding one angle of a triangle if the other two are known.

Exercises

Solve the following exercises using the formula $a+b+c=180$.

1. Make a triangle so that the first angle is three times the second, and the third is six times the second.

Solution: Let x be the number of degrees in the second angle.

Then $3x$ is the number of degrees in the first, and $6x$ is the number of degrees in the third.

$x+3x+6x=180$, for the sum of the angles of a triangle is 180°.

Hence, $10x=180$, by combining similar terms.

$x=18$, by dividing both sides of the equation by 10.

$\therefore 3x=54$,

and $6x=108$.

Check: $x+3x+6x=180$.

2. The three angles of a given triangle are equal. Find them.

Suggestion: Let x be the number of degrees in each angle, form an equation containing x, and solve.

3. The first angle of a triangle is twice as large as the second, and the third is 3 times the first. Find the value of each.

Suggestion: Let x be the number of degrees in the second angle.

4. Two angles of a triangle are equal and the third is equal to the sum of the other two. Find each angle.

5. The first angle of a triangle is four times as large as the second, and the third is one-half the first. Find each angle.

6. Find the angles of a triangle if the first is 6 times as large as the second, and the third one-half as large as the first.

7. The first angle of a triangle is one-half as large as the second, and the third is three-fourths as large as the second. Find each angle.

Solution: Let n be the number of degrees in the second angle.

Then $\frac{1}{2}n$ is the number of degrees in the first,

and $\frac{3}{4}n$ is the number of degrees in the third.

Then $n+\frac{1}{2}n+\frac{3}{4}n=180$

$\therefore \frac{9}{4}n=180$

$\therefore n=80$, by dividing both members by $\frac{9}{4}$

$\frac{1}{2}n=40$

$\frac{3}{4}n=60$

8. The first angle of a triangle is one-third of the second, and the third is one-half of the first. Find the three angles.

57. Measurement of angles in surveying. A simple instrument for measuring angles out of doors may be made by a pupil as follows:

On a drawing board (Fig. 98) fasten a large protractor. By means of a pin stuck into the board at O

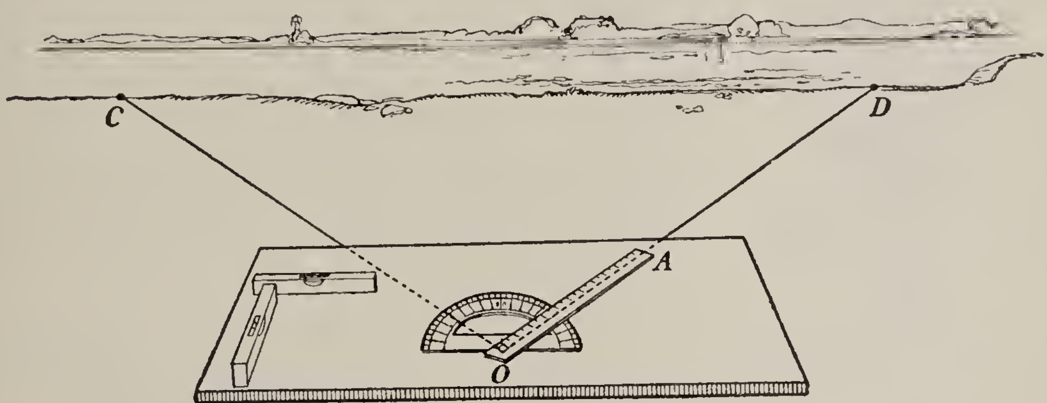


FIG. 98

attach a ruler so that it may move freely over the board when it is made to turn about O. A pin stuck in at A near the other end of the ruler may be used for sighting.

The board may be placed on a tripod, or a table, and brought into horizontal position by means of two spirit levels attached as shown in the figure.

To measure $\angle DOC$, sight in the direction of the side OD and take the reading on the protractor. Repeat this for the side OC. The difference between the two readings is the *measure* of the angle.

Surveyors and astronomers measure angles with a transit (Fig. 99). The telescope on this instrument is used for *sighting* along the sides of the angle which is to be measured. The *measure* of the angle is found by

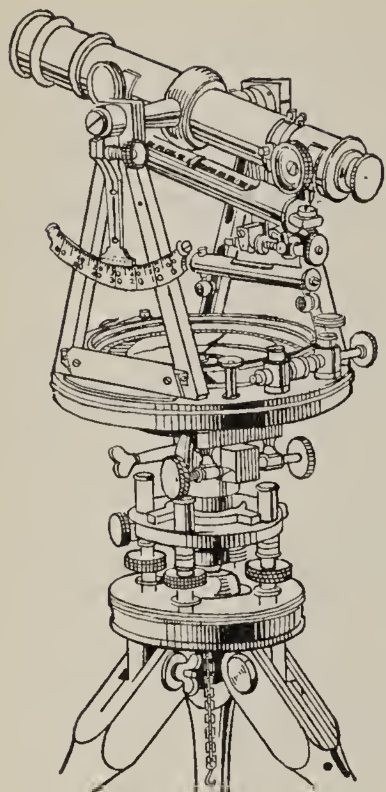


FIG. 99
SURVEYOR'S TRANSIT

means of two graduated circles which serve as protractors. One of these is used to measure *vertical* angles, as AOB (Fig. 100) and the other for measuring *horizontal* angles, as COD (Fig. 98).

58. Measurement of angles in navigation. By measuring angles the navigator is able to avoid dangerous obstacles, as rocks or sandbanks (Fig. 103), which lie near the course of his ship. He uses for this

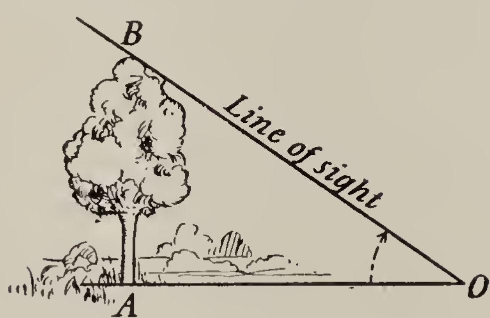


FIG. 100

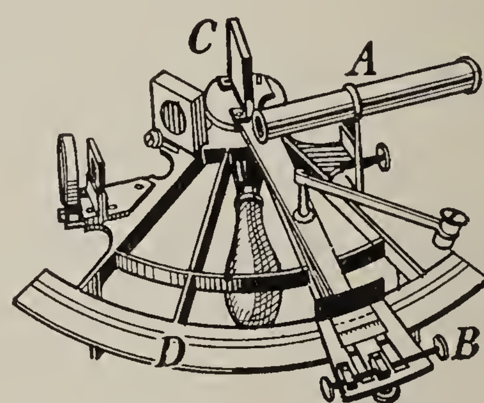


FIG. 101. SEXTANT

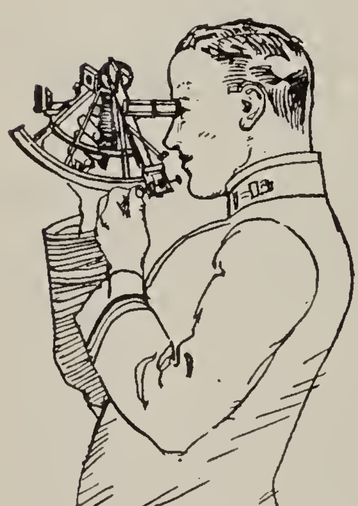


FIG. 102
MEASURING AN ANGLE WITH A SEXTANT

an instrument called the sextant (Fig. 101).

The instrument is used to measure an angle whose vertex is the observer's eye and whose sides pass through 2 remote objects. It is held by the handle in the right hand, with the telescope A (Fig. 101) toward the observer's eye (Fig. 102). Making the line of sight pass

through one of the objects, the observer moves the sliding arm B until the image of the second object is reflected by the mirror C into the telescope. The angle is found by means of the scale on the arc D of the sextant.

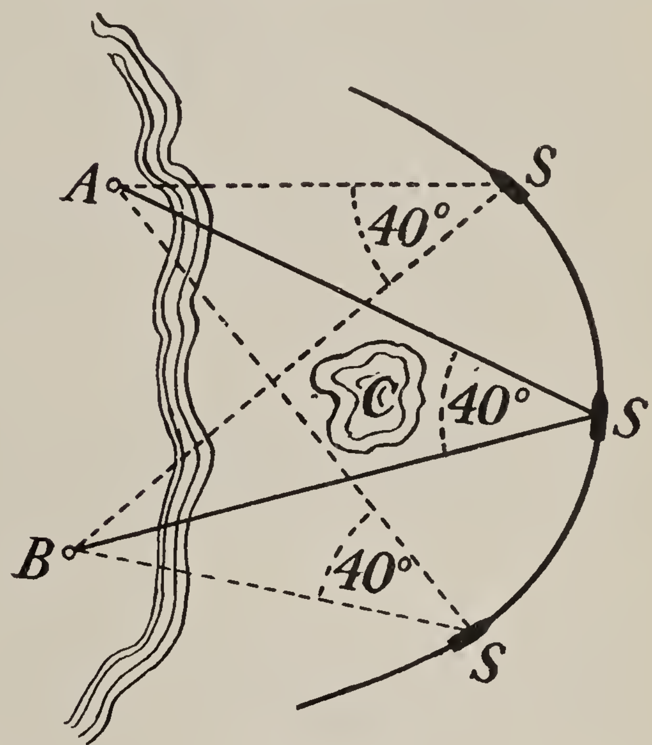


Fig. 103

As the ship S (Fig. 103) approaches the dangerous region C, two well defined objects A and B on the shore line are constantly observed from the ship with the sextant. This gives repeated measures of angle ASB. The ship must round the obstacle in such a way that angle ASB does not increase in size.

Fig. 104. Mariner's Compass.

The *mariner's compass* (Fig. 104) is used to show the *direction* (angle between the course of

the ship and the north-south line) in which a ship is traveling.

The rim of the compass is divided into 32 equal parts, called *points*, each of which has a name as shown in the picture.

State the size of the angles formed by the following directions: *E* and *SE*, *NE* and *W*, *N* and *E*, *SE* and *NW*, *WSW* and *E*, *S* and *NW*.

59. Adjacent angles. Two angles ABC and DEF (Fig. 105) may be placed so that the vertices *E* and *B* coincide (Fig. 106) and that side *ED* falls along side *BC*. In this position the angles are said to be *adjacent* to each other. In general, two angles are **adjacent** if they have a *common* (the same) vertex and a common side between them.

The other two sides, *BA* and *BF*, are called *exterior* sides.

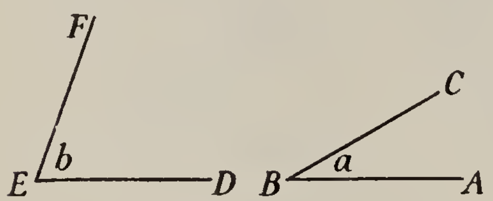


Fig. 105

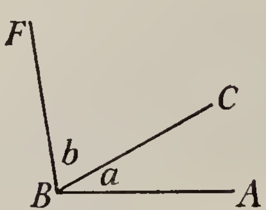


Fig. 106

Exercises

1. State whether or not the following angle pairs (Fig. 107) are adjacent and give reasons for your answers: *a* and *b*, *b* and *c*, *c* and *d*, *d* and *e*, *e* and *f*, *f* and *g*.

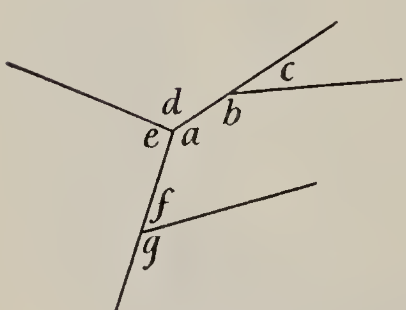


Fig. 107

2. Make sketches showing two acute adjacent angles; two obtuse adjacent angles.

3. Find adjacent angles in the classroom.

Fig. 108

4. Measure the adjacent angles m and n (Fig. 108).

5. Draw two intersecting lines making one of the adjacent angles 4 times as large as the other. Begin by drawing a sketch. Then find the angles by means of an equation.

Solution: Let x be the number of degrees in one angle.
Then $4x$ is the number of degrees in the adjacent angle,
and $4x+x=180$.

Solve this equation.

Make an *accurate* drawing.

6. One of two adjacent angles formed by two intersecting lines is 3.5 times as large as the other. How large is each angle?

7. One of two adjacent angles is $\frac{1}{5}$ as large as the other. Find the number of degrees in each.

60. Perpendicular lines. Two straight lines forming *equal adjacent* angles are **perpendicular** to each other. Thus the equation $m=n$ (Fig. 109) states in symbols that HF is perpendicular to EG. Another symbol for perpendicularity is $\perp$. Hence, the statement *HF is perpendicular to EG* may be written briefly $HF \perp EG$.

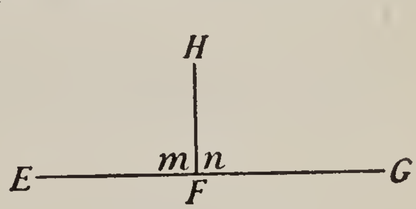


FIG. 109

EXERCISES

1. State in the two ways explained above that the lines in Fig. 110 are perpendicular to each other.


FIG. 110

2. Two line segments may be perpendicular to each other without intersecting. Which of the lines in Fig. 111 are perpendicular? Give reasons.

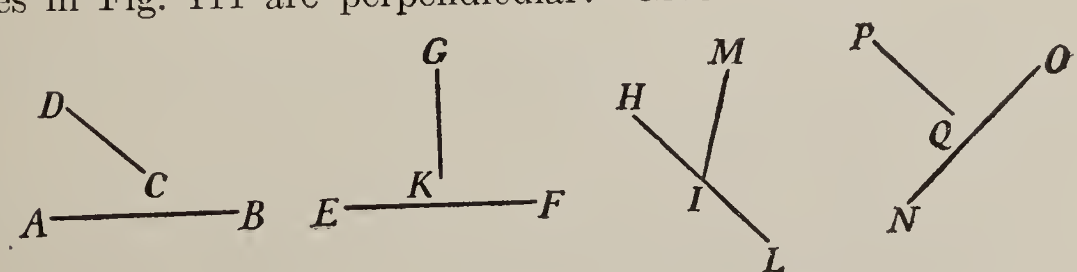


FIG. 111

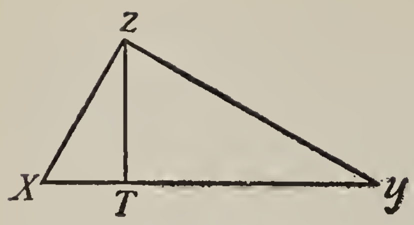


FIG. 112

3. Locate perpendicular lines in the classroom; in Fig. 112.

4. Prove by means of an equation that if two lines are perpendicular to each other the two adjacent angles are right angles (Fig. 113).

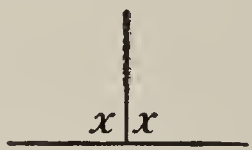


FIG. 113

5. Exercise 4 suggests the following use of a triangle having one right angle for drawing a line perpendicular to a given line to AB (Fig. 114) and passing through a given point C.

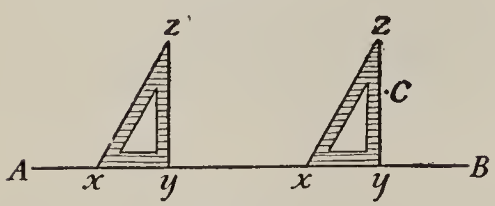


FIG. 114

Directions: Place a right triangle XYZ so that one side of the right angle lies on AB. Slide the triangle along AB until YZ passes through C. Then draw a line along YZ. This is the required line.

61. Supplementary angles. Measure angles m and n (Fig. 115) and find the sum. Two angles whose sum is 180°, or a straight angle, are **supplementary angles.** Each is said to be the *supplement* of the other.

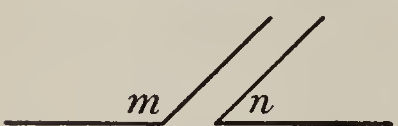


FIG. 115

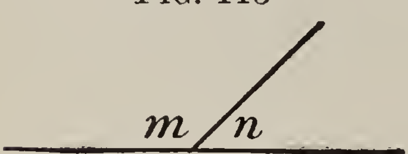


FIG. 116

When two supplementary angles are *adjacent* (Fig. 116) they are called *adjacent supplementary angles.*

EXERCISES

1. Using only a ruler, draw a sketch of two supplementary angles. Measure with a protractor each of the angles and test the accuracy of your drawing.

2. How many degrees are there in the supplement of 45°? of 20°? of 160°? of 130°? Tell how the supplement is found in each case.

3. Find the supplement of $67° 15'$.

Solution: $180° = 179° 60'$.
Subtracting $67° 15'$,
we have the supplement $= 112° 45'$.

4. Find the supplement of $110° 30'$; of $25° 40'$; of $18° 57'$; of $90° 40' 32''$, arranging your work as in Exercise 3.

5. Find the supplement of $a°$; of $x°$. Write the result in the form of binomials.

6. Make a formula for finding the supplement, s, of any given angle, a.

7. State by an equation that $a°$ and $b°$ are supplementary.

8. State by equations that the following pairs of angles are supplementary

$x°$ and $50°$; $x°$ and $\frac{1}{2}x°$; $(x+20)°$ and $(2x-4)°$.

9. One of two supplementary angles is 5 times as large as the other. Find the two angles by means of an equation.

10. One of two supplementary angles is .8 as large as the other. Find the two angles.

11. Find two supplementary angles, one of which is $1\frac{4}{7}$ times as large as the other.

62. Complementary angles. Measure angles x and y (Fig. 117) and find the sum.

Two angles whose sum is $90°$, or a right angle, are **complementary angles.** Each is the *complement* of the other.

FIG. 117

EXERCISES

1. Make a sketch of two complementary angles. Test your drawing with a protractor.

2. Make a sketch of two adjacent complementary angles.

3. Arranging your work as in Exercise 3 (§61) find the complement of 30°; 70°; 80.5°; 27° 14′; 18° 25′; 16° 13′ 40″; 65° 25′ 32″.

4. Find the complement of $a°$; $x°$. Write the results as binomials.

5. Make a formula for finding the complement, c, of a given angle a.

6. State by means of an equation that 20° is the complement of $a°$; that $x°$ is the complement of $\frac{5x°}{4}$.

7. One of two complementary angles is 8 times as large as the other. Find the angles by using an equation.

8. A right angle is to be divided into two parts so that one is $5\frac{1}{2}$ times as large as the other. Find the two parts.

9. Draw a right triangle (a triangle having a right angle). Show by measuring that the acute angles are complementary. Naming the acute angles a and b, state the equation.

10. Find the acute angles of a right triangle if one is 3 times the other; $\frac{2}{3}$ as large as the other. Make sketches of the angles.

63. Opposite angles. Draw two intersecting lines, as AB and CD (Fig. 118). Measure the angles m, n, r, and s. In each angle write the number of degrees it contains. How do angles m and r compare as to size? Compare angles n and s.

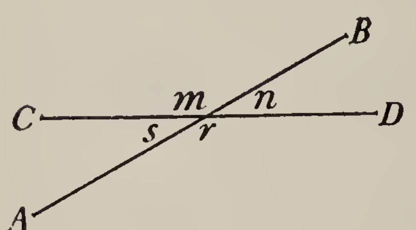


Fig. 118

State by means of equations the relations between m and n; m and s; s and r; r and n. What are these angle pairs called?

State the relations between m and r; s and n.

The angle-pairs m and r, n and s are called *opposite*,

or *vertical,* angles. **Opposite angles** are formed by two intersecting straight lines so that the sides of one angle lie in the same straight lines as the sides of the other, but in opposite directions from the vertex.

From the measures of the angles (Fig. 118) it is seen that *if two lines intersect, the opposite angles are equal.*

Exercises

1. Draw two intersecting straight lines and name the opposite angles. Express by equations the fact that the opposite angles are equal.

2. Two intersecting lines make one angle equal to 32°. Find the other angles.

3. Find the sum of the angles just covering the plane surface around a point (Fig. 119). Express the result in the form of an equation.

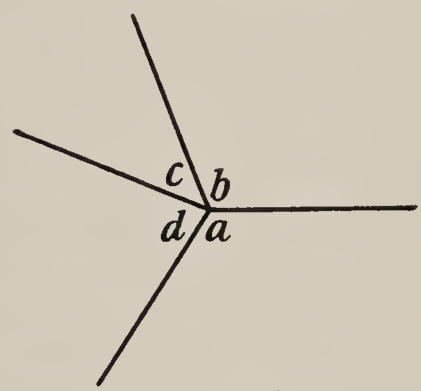


Fig. 119

64. Summary of angle relations. Express the relations for the following angles in the form of equations; in words; and by means of figures.

1. The angles of a triangle.

2. Two supplementary angles.

3. Two complementary angles.

4. Two opposite angles.

5. The acute angles of a right triangle.

6. The adjacent angles formed by two perpendicular lines.

Drawing Angles with the Protractor

65. To draw an angle of a given size. To draw an angle of 45°, draw first a straight line, as *ABC* (Fig. 120).

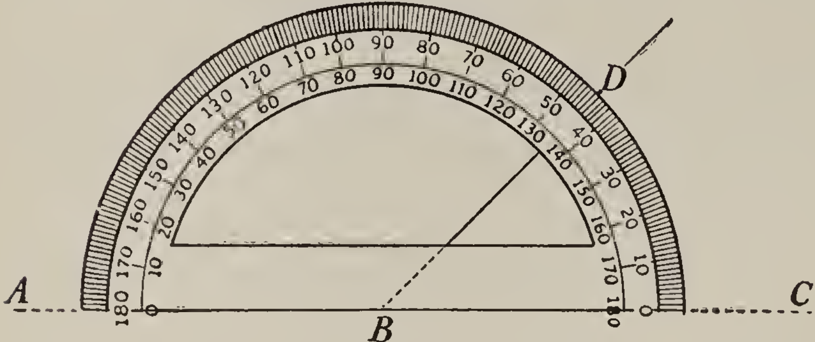


Fig. 120

Then place the center of the protractor at *B* and the zero-mark exactly on *BC*.

Starting at the zero-mark, pass along the rim and place a point *D* at the 45° mark.

Remove the protractor and draw a line from *B* passing through *D*.

Angle *CBD* is the required angle.

Exercises

1. Using the protractor and straightedge, draw an angle equal to 30°; 90°; 120°; 180°; $65\frac{1}{2}°$; $94\frac{1}{2}°$.

2. By means of the protractor draw a line perpendicular to a given line *BC*, at one of its points, as *A*.

3. Draw a triangle having a right angle.

66. To draw an angle equal to a given angle. In making designs like those of Fig. 121 one must be able to draw angles of the same size as the angles in the required design.

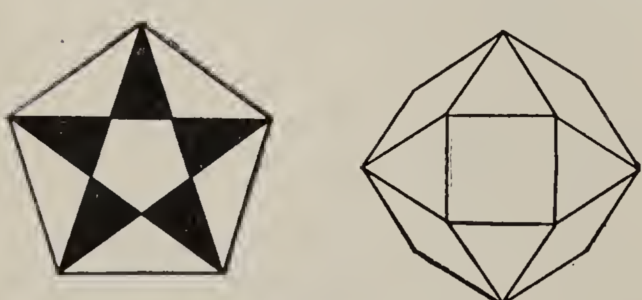


Fig. 121

This may be done by means of the protractor. For example, let it be required to draw an angle equal to $\angle ABC$ (Fig. 122). Measure $\angle ABC$ and write the number of degrees it contains inside of the angle, near the vertex.

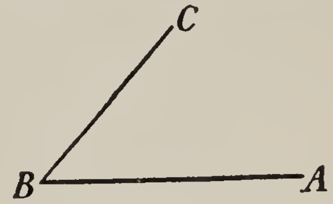


FIG. 122

Draw a line as DEF (Fig. 123), and by the method of §65 draw on DF at the point E an angle containing the same number of degrees as ABC.

FIG. 123

This is the required angle.

EXERCISES

1. With a protractor and ruler draw a triangle having one right angle. What is the relation between the acute angles? Express the result in the form of an equation.

2. Using the protractor to draw the right angles, draw a square whose sides are 3 cm. long.

3. Draw a rectangle (Fig. 9) having two consecutive sides equal to 2″ and 4″ respectively.

4. Draw two intersecting lines, as AB and CD (Fig. 124). Mark a point E on CD. At E draw an angle x' equal to x.

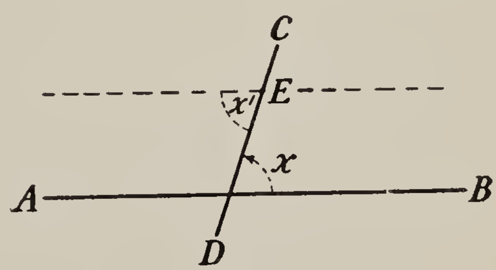


FIG. 124

5. Make the designs shown in Fig. 121.

6. Draw a triangle having two equal angles. To test the accuracy of your drawing measure the two sides opposite the equal angles. Compare them as to length.

This exercise shows that *if two angles of a triangle are equal the sides opposite them are equal.*

7. In the making of trusses for bridges, the beams are put together in the form of triangles having two equal sides. Make the

design (Fig. 125). Lay off $AB = BC = CD$. Then draw angles equal to 60 degrees at A, B, C, and D.

If your drawing is well made, the triangles AEB, BFC, and CGD have three equal sides, and the points E, F, G lie on the same straight line.

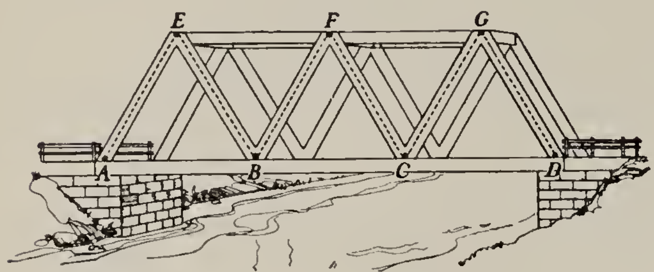


Fig. 125

8. In a triangle two angles are equal. The side opposite one of them is 6 inches long. How long is the side opposite the other?

9. Draw a triangle having each of two angles equal to 60 degrees. What must be the size of the third angle? Measure the third angle and thus test the accuracy of your drawing. Measure the sides of the triangle.

This exercise shows that *if the three angles of a triangle are equal, the three sides of the triangle are equal.*

10. Draw a triangle having one angle equal to 60 degrees and another equal to 30 degrees. What is the size of the third angle?

Measure to two decimal places the side opposite the 90-degree angle, and the side opposite the 30-degree angle. Find the ratio of the two sides.

The exercise shows the following: *If in a right triangle the acute angles of which are 30 degrees and 60 degrees, the side opposite the 90-degree angle is twice as long as the side opposite the 30-degree angle.*

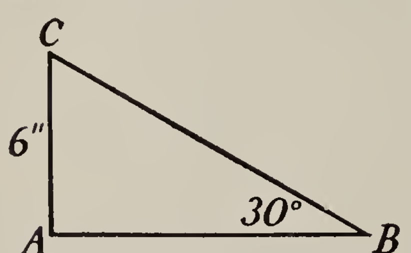


Fig. 126

11. In the right triangle ABC (Fig. 126) the side opposite the 30-degree angle is assumed to be 6 inches long. What is the length of CB?

67. Isosceles triangle. Equilateral triangle. Right triangle. A triangle having *two* equal sides is an **isosceles** triangle. A triangle having *three* equal sides is **equilateral.** A triangle having a *right* angle is a **right triangle.** The side opposite the right angle is called the **hypotenuse.**

Parallel Lines

68. Meaning of parallel lines. Draw a line segment AB (Fig. 127) about 12 cm. long. Place the sharp points of your compass on the ruler, or squared paper, 8 cm. apart. On AB lay off segment CD equal to 8 cm. in length.

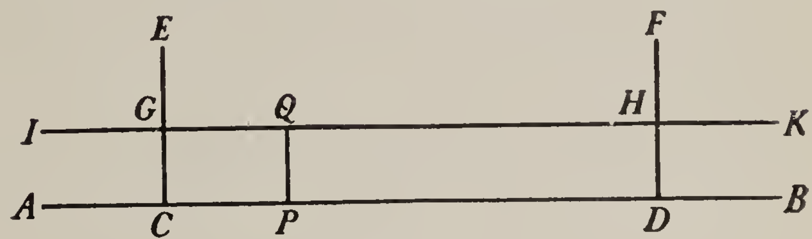


Fig. 127

At C and D draw lines CE and DF perpendicular to AB, using the protractor.

On CE and DF lay off two equal lengths, as CG and DH.

Draw line IK passing through G and H.

Line IK is said to be *parallel* to AB.

Measure angles DHK and CGI and show that

CG and DH are perpendicular to IK.

Select any point P on AB. Draw a line perpendicular to AB at P and denote the point where it intersects IK by Q. Measure PQ.

How does the length of PQ compare with that of CG; of DH?

Because the perpendicular PQ which was drawn at a point P selected *anywhere* on AB, or its extension, is equal to the fixed lengths CG or DH, the lines AB and IK are said to be *everywhere* equally far apart. Hence, *they cannot meet however far they are extended.*

If two lines are drawn in the same plane surface, and if they do not intersect, however far extended, they are

called **parallel lines.** The word *parallel* means *running alongside of each other.*

By the *distance* between two *parallel lines* is meant the length of the perpendicular between them.

One of the properties of parallel lines is that they are everywhere equally distant.

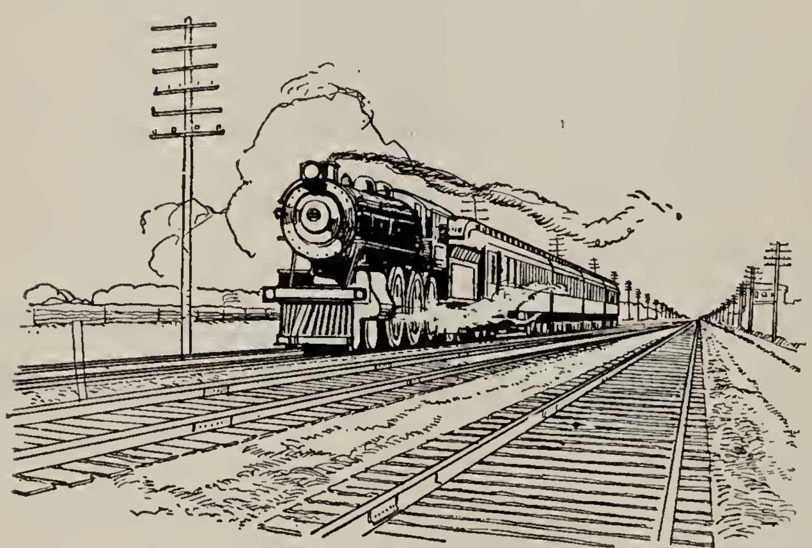
69. Symbol for parallelism. The statement *AB is parallel to CD* is written briefly in symbols: $AB \parallel CD$.

Exercises

1. Point out parallel lines in the classroom.

2. On a cube, or on a rectangular block, point out parallel lines.

3. Point out parallel lines on squared paper; on the ruler.



4. On a cube point out two lines which do not meet and are not parallel.

5. In the classroom point out two lines which do not meet and are not parallel.

6. Are the rails in the adjoining picture parallel? Give reason for your answer. Do they look parallel to you?

7. Considerable knowledge of parallel lines was developed by the primitive races in connection with the art of weaving, basketry, and pottery. Many of the decorative designs are based on parallel lines, taking the shapes of rectangles, parallelograms, and squares (Fig. 9). Try to collect designs in weaving, clothing, pottery, household implements, etc., which illustrate the use of parallel lines.

70. Drawing parallel lines. On squared paper draw two parallel lines AB and CD (Fig. 128) and a line EF intersecting AB and CD.

Measure angles a and b.

How do they compare as to size?

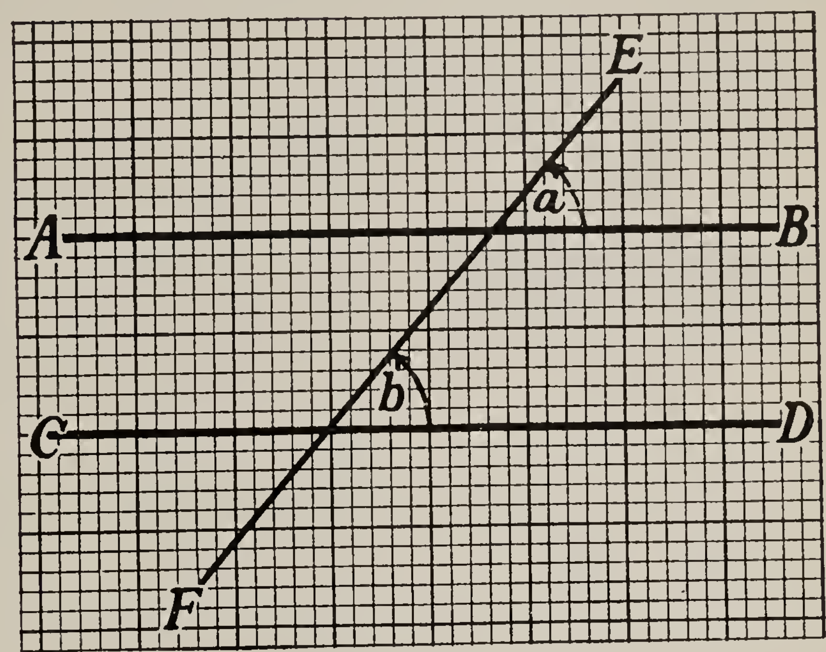


FIG. 128

The equality of angles a and b (Fig. 128) suggests the following methods of drawing parallel lines.

1. *The triangle method.* Place one side, AB, of the triangle ABC (Fig. 129) along EF. Draw a line along the side BC.

Move the triangle by sliding side AB along EF until it takes the position $A_1\ B_1\ C_1$. Draw a line along $B_1\ C_1$.

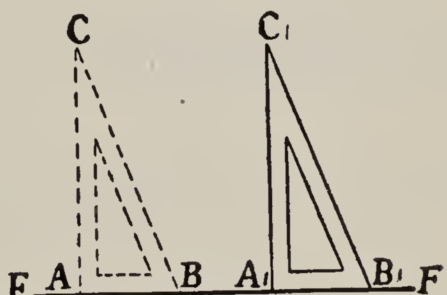


FIG. 129

Then BC is parallel to $B_1\,C_1$ because the angles at B and B_1 are equal.

2. *The T-square method.* Place the head of the T-square (Fig. 130) along one edge AB of the drawing board. Draw a line along the straight edge. Then move the head of the square downward

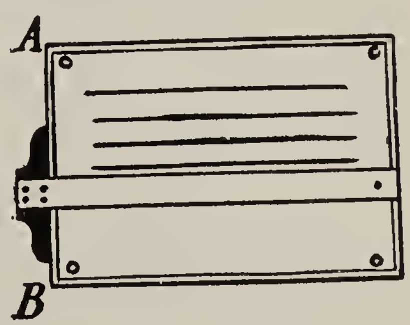


FIG. 130

along AB to the position of the desired parallel line, and draw a second line along the straight edge. Why are these lines parallel?

3. *The carpenter's square method.* Move one side of the square along an edge of the board (Fig. 131) and draw lines along the other side.

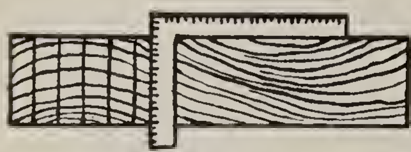


FIG. 131

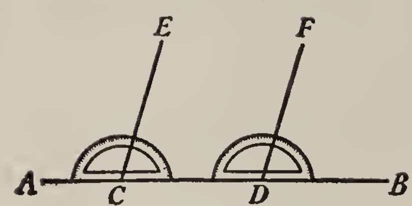


FIG. 132

4. *The protractor method.* Draw line CE (Fig. 132) and measure angle DCE.

At D draw angle BDF equal to angle DCE.

Then CE is parallel to DF. Why?

5. *The parallel ruler method.* Draftsmen and navigators use a *parallel ruler* (Fig. 133) for drawing parallel lines. Two rulers are connected with cross pieces of brass, which work on pivots in such a way that the rulers may be spread apart or brought together, always remaining parallel to each other. This instrument is usually made of ebony.

FIG. 133

EXERCISES

1. A **parallelogram** is a quadrilateral whose opposite sides are parallel. On unruled paper draw a parallelogram whose adjacent sides are 4 cm. and 6 cm., respectively, including an angle of 40°.

Suggestion: Make the opposite sides parallel by using the protractor method (§70).

Measure the four sides. How do they compare as to size?

2. A **rectangle** is a parallelogram whose adjacent sides are at right angles to each other. Draw a rectangle having two adjacent sides equal to 3 cm. and 5 cm., respectively.

3. A **square** is an equilateral rectangle.
Draw a square whose side is 5 centimeters.

71. Angle pairs formed by three lines. If two lines AB and CD (Fig. 134) are intersected by a third line, eight angles are formed. These may be grouped in pairs as follows:

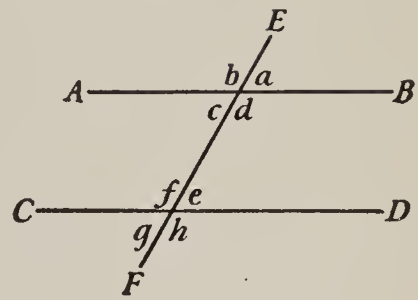


FIG. 134

Opposite angles: a, c; b, d; e, g; f, h.

Adjacent supplementary angles: a, b; b, c; c, d; d, a; e, f; f, g; g, h; h, e.

Corresponding angles: a, e; b, f; c, g; d, h.

Interior angles on the same side of EF: c, f; d, e.

Alternate interior angles: c, e; d, f.

Alternate exterior angles: a, g; b, h.

EXERCISES

1. It has been shown (§63) that opposite angles are equal. State the equations for angles

a and c, b and d, e and g, f and h.

2. State the equations for the adjacent angles

a and b, b and c, c and d.

3. If AB is parallel to CD, state the relations between a and e, d and h, b and f, c and g (§70). This may be stated as a geometric principle as follows:

If two parallel lines are cut by a transversal (cutting line), *the corresponding angles are equal.*

4. The methods of drawing parallel lines (§70) are based upon the principle that *two lines are parallel if the corresponding angles formed with a transversal are equal.*

We shall now *prove* that the *alternate interior* angles are equal if the *corresponding* angles are equal.

Let $a=e$.

We know that $a=c$. Why?

Hence we can replace in the first equation the number a by its equal c. This gives $c=e$.

5. Prove that $f=d$ when $b=f$.

6. Prove that $d=h$ if $f=d$.

7. Prove that *if the alternate interior angles formed by two lines and a transversal are equal the lines are parallel.*

Proof: If the alternate interior angles are equal, the corresponding angles are equal.

If the corresponding angles are equal the lines are parallel.

$\therefore$ If the alternate interior angles are equal the lines are parallel.

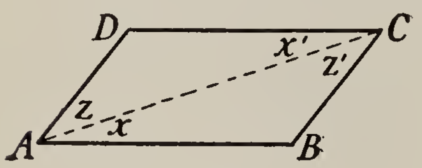


FIG. 135

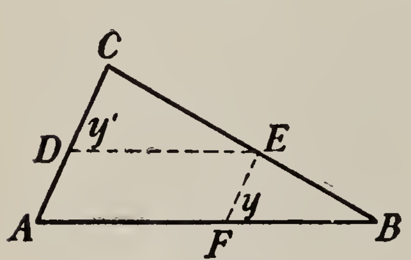


FIG. 136

8. One of two alternate interior angles formed by two parallel lines and a transversal is 66 degrees. The other is denoted by $11x$. What is the value of x?

9. Prove that in the parallelogram $ABCD$ (Fig. 135) the angles x and x' are equal. Prove that z and z' are equal.

10. Line DE (Fig. 136) is parallel to AB, and EF is parallel to CA. Prove that $y=y'$.

72. What every pupil should know and be able to do. The pupil should understand the meaning of the following terms: angle, vertex, side; right, acute, obtuse, straight angle; adjacent, complementary, supplementary angles; perpendicular, parallel lines; isosceles, equilateral triangle, right triangle, hypotenuse.

Every pupil should know how to do the following:

1. To use the protractor to measure and draw angles; to draw perpendicular and parallel lines.

2. To solve equations of the forms $x+25=90$, and $6x+x+3x=180$.

3. To solve problems leading to equations of the form given in 2.

The following principles should be known:

1. *The sum of the angles of a triangle is 180°.*

2. *If two lines intersect, the opposite angles are equal.*

3. *The acute angles of a right triangle are complementary.*

4. *If two angles of a triangle are equal, the sides opposite them are equal.*

5. *If three angles of a triangle are equal, the triangle is equilateral.*

6. *In a right triangle with acute angles equal to 30 degrees and 60 degrees, the hypotenuse is twice as long as the side opposite the 30-degree angle.*

7. *If two parallel lines are cut by a transversal the corresponding angles are equal; the alternate interior angles are equal.*

The pupil should know the table of angular measurement, and be able to change degrees to minutes and seconds.

73. Typical problems and exercises. Every pupil should be able to answer questions and solve problems of the types given below.

1. Classify the following angles (Fig. 137).

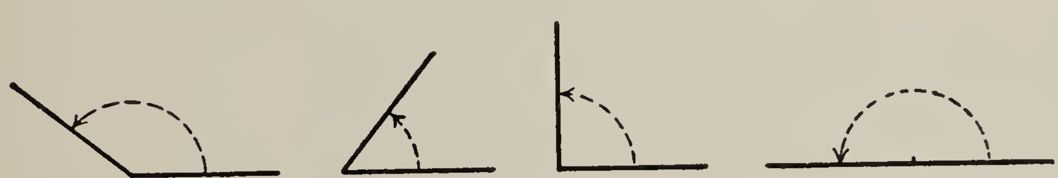


FIG. 137

2. If the first angle of a triangle is twice as large as the second, and if the third angle is 6 times as large as the second, how large is each angle?

3. Draw a line perpendicular to a given line and passing through a point not on the given line.

4. Draw a line parallel to a given line and passing through a point not on the given line.

5. Change 18° 14′ 12″ to seconds.

6. One of two supplementary angles is twice as large as the other. How large is each?

7. Three angles just cover the plane surface around a point. They are denoted by x, $2x$, and $6x$. Find the number of degrees in each.

8. Draw a triangle two of whose angles are 40 degrees and 80 degrees, respectively.

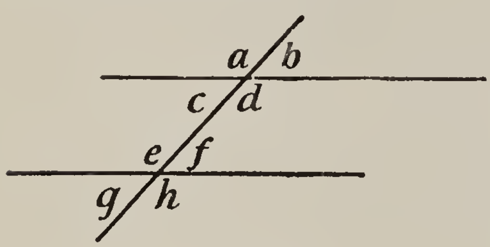


FIG. 138

9. State the relations for the angle pairs in Fig. 138.

10. Write a paper on one of the following topics:

a. Uses of angles in designs; in building; in navigation; etc.

b. Historical development of the angular unit.

c. Parallel lines.

CHAPTER V

USES OF LINE SEGMENTS AND ANGLES IN FINDING UNKNOWN DISTANCES

The Congruent-Triangle Method of Finding Distances

74. How to measure line segments indirectly. We have been studying line segments, angles, and some facts about triangles. We are now prepared to make a more extensive study of the triangle.

The triangle is used in ornamental work (Figs. 139, 140); in designing (Fig. 141); in construction (Fig. 142); in surveying (Fig. 143); in navigation (Fig. 144).

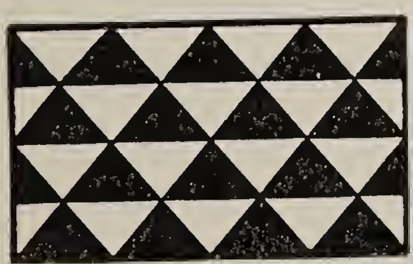


Fig. 139. Tile Flooring

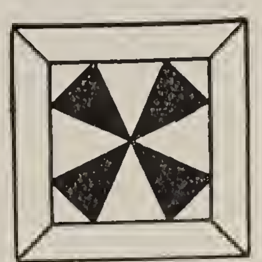


Fig. 140. Parquet Flooring

Fig. 141. Design for a Church Window

FIG. 142. USE OF A TRIANGLE IN THE CONSTRUCTION OF A BRIDGE

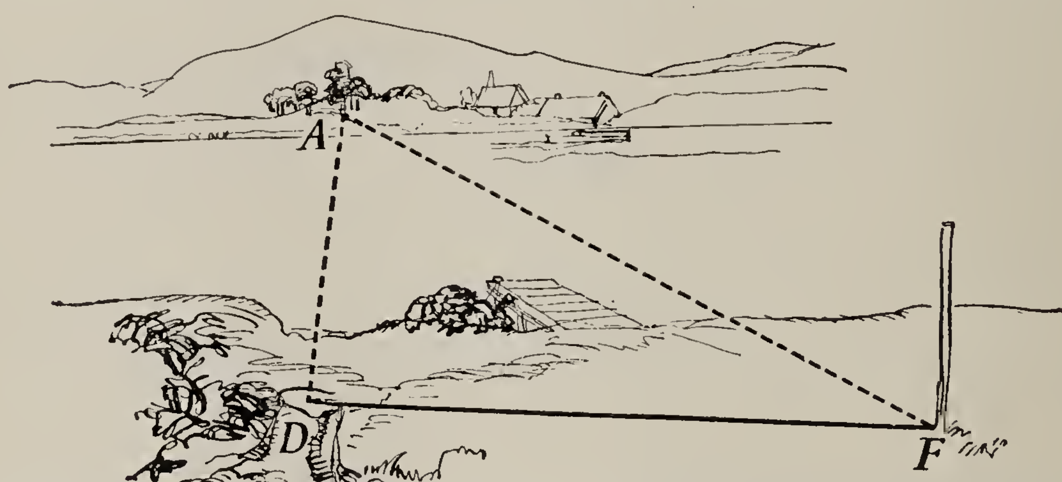


FIG. 143. SURVEYOR'S CHART FOR FINDING THE DISTANCE ACROSS A STREAM, *AD*

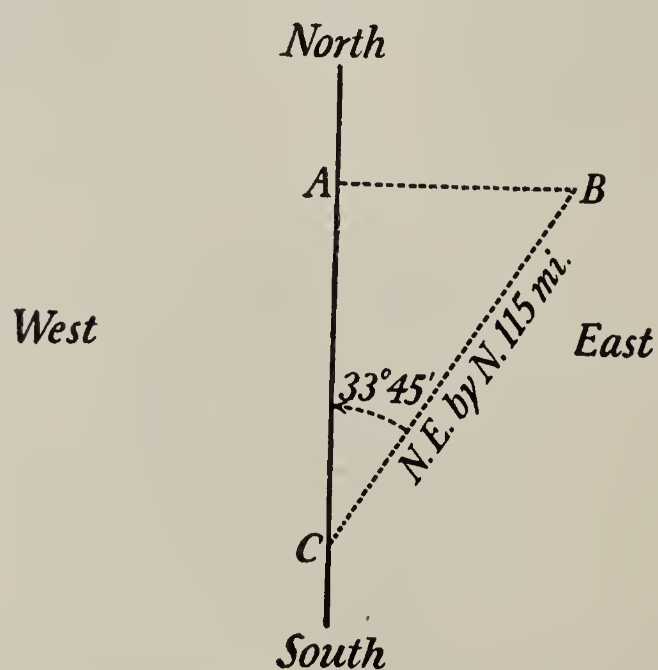


FIG. 144. TRIANGLE FOR DETERMINING THE EAST AND NORTH DISTANCES MADE BY A SHIP

In this chapter we shall learn to use the triangle to determine distances which cannot conveniently be measured directly, *e.g.*, the height of a tree (Fig. 145), the distance across a river (Fig. 146); or those which cannot be measured directly at all, as the distance through a building or a hill (Figs. 147, 148).

Fig. 145

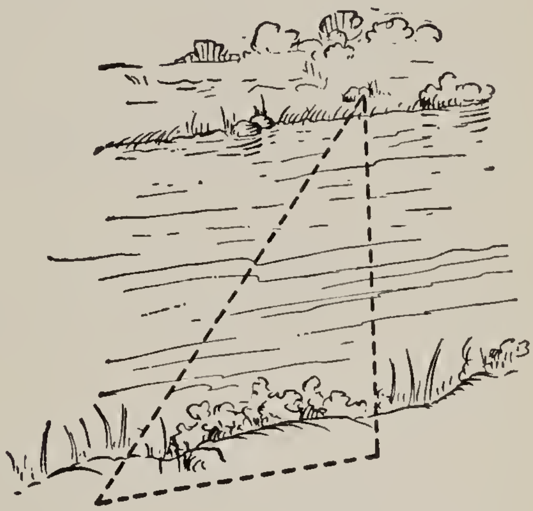


Fig. 146

Fig. 147

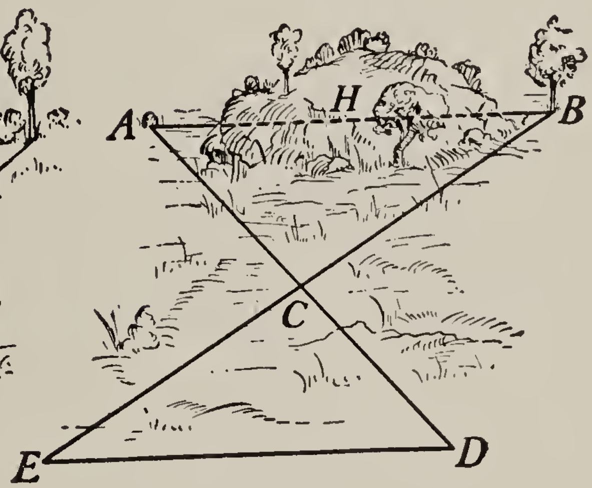


Fig. 148

To be sure, the height of a tree can be determined by climbing the tree and then applying a tape line. The distance across a river can be found by swimming across and measuring a line stretched from one bank to the other. A knowledge of mathematics, however,

makes this unnecessary. It saves work and time. Furthermore, by mathematics we can solve the problem even when direct measurement is impossible, as in Figures 147 and 148.

For example, in Fig. 148, to find the distance AB passing through the obstructing hill H, a point C is selected so that lines BE and AD may be laid off conveniently. A triangle ECD of the same size and shape as triangle ACB is laid off by making $CE = CB$, and $CD = CA$. The length of AB is then the same as that of ED, and can therefore be found by measuring ED.

Another method of determining AB makes use of a triangle of the same *shape* as ABC, but not of the same *size*. A third method uses a *right triangle*. All of these methods obtain the required length *without measuring the unknown line directly*. Determining distances without measuring directly is called **indirect measurement.**

75. Triangles of the same size and shape. In the problem of finding unknown distances it is necessary to know how to make a triangle which is exactly of the same size and shape as another triangle. One triangle is then an exact reproduction of the other. The two triangles are really the same triangle in two different positions, and one can be made to fit exactly on the other. Such triangles are called **congruent** triangles. This word comes from the Latin word *congruere*, meaning *to agree*. Congruent triangles have the sides and angles of one, equal to the corresponding sides and angles of the other. The symbol for congruence is $\cong$, the symbol $=$ meaning equal in size, and $\backsim$ meaning similar in shape.

Examples of congruence are the "blue prints" of the draftsman, the reprints of an original, the "negative" plate in photography.

Exercises

1. The story is told that a soldier of Napoleon was commanded by him to determine the width of a river which the army had to cross. In a brief time he brought in the desired information. When asked how he had obtained his result he said that he did it by means of mathematics as follows: Standing at the point S (Fig. 149) and

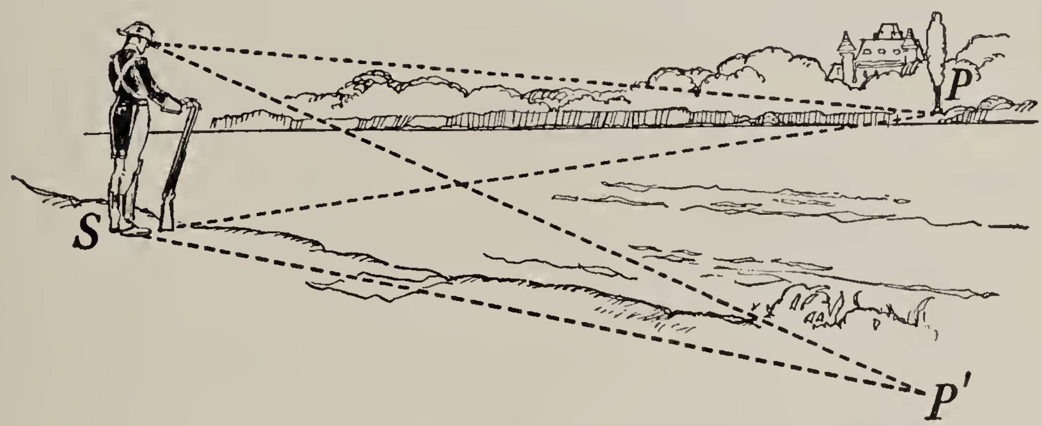


Fig. 149

looking across the river, he lowered his head until a point P on the opposite bank was exactly in line with the rim of his hat and his eye. Keeping his head in rigid position, he turned, sighted along the shore line, and had a stake placed at P', a point on the shore, exactly in line with his eye and the rim of his hat.

SP' was then measured and the required distance SP determined. Explain why $SP' = SP$.

2. Draw a line AB (Fig. 150) of indefinite length.

With the compass, lay off on AB a distance $AC = 8$ cm.

On AB at A draw an angle equal to 50 degrees.

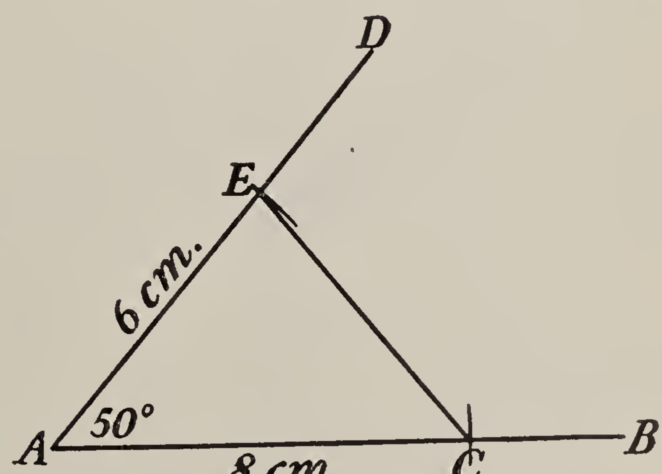


Fig. 150

On AD lay off $AE=6$ cm.

Draw EC.

Cut triangle ACE from the paper, write your name on it, and pass it to the teacher.

By placing together all the triangles made by the pupils of the class, it is seen that they can be made to coincide. If all the drawings are exact, they will all fit exactly.

3. Draw a triangle ABC. Measure AB, AC, and angle CAB. As in Exercise 2, draw a second triangle with two sides equal to AB and AC, and with the angle included between these sides equal to angle CAB. Place the first triangle on the second. If your drawing is well-made, the two triangles can be made to fit exactly.

76. The congruent-triangle method of finding unknown distances. Exercises 2 and 3 show that two triangles can be made to fit exactly if they have two sides of one equal to two sides of the other, and the included angles equal. These exercises illustrate a fact which is usually stated as follows:

Two triangles are **congruent** *if two sides and the included angle of one are equal respectively to two sides and the included angle of the other.*

Exercises

1. A surveyor may use the principle explained above to find unknown distances as is shown in the following:

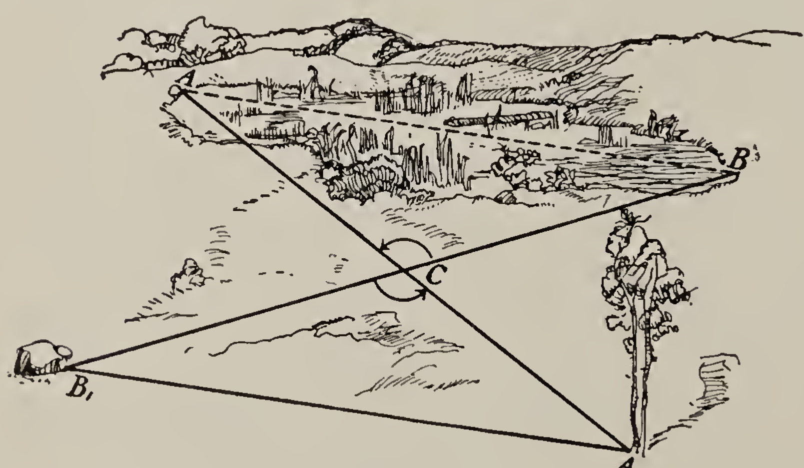


Fig. 151

Let AB (Fig. 151) be the unknown distance across a swamp, and let it be required to measure AB.

A suitable point C is selected so that lines may be drawn from A and B through C.

CB_1 is laid off equal to CB, and CA_1 equal to CA.

Then $A_1 B_1$ is drawn.

Show that triangle $ABC \cong$ triangle $A_1 B_1 C$.

Show that the length of AB may be found by measuring $A_1 B_1$, giving reasons for your statements.

2. Tell how to find an unknown distance by the congruent-triangle method.

3. Draw a line AB (Fig. 152) of indefinite length.

Open the compass a distance of 8 cm. between the sharp points.

On AB lay off $AC = 8$ cm.

At A on AC draw an angle equal to 40 degrees.

At C draw an angle equal to 70 degrees.

Cut triangle ACD from the paper, write your name on it, and pass it to the teacher.

By placing the best drawn triangles together it will be seen that they coincide.

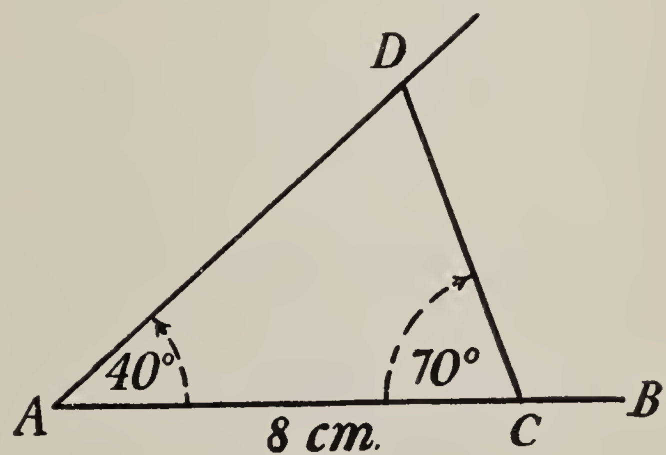


Fig. 152

Note that two angles and the side between their vertices in one triangle, are equal respectively to two angles and the side included between the vertices in the other triangle.

4. Draw a triangle ABC (Fig. 153). Draw a second triangle having two angles equal to angles A and B, and the side included between the vertices equal to AB.

Cut the second triangle from paper and see if you can fit it on $\triangle ABC$.

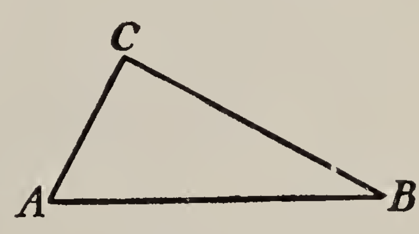


Fig. 153

Exercises 3 and 4 illustrate the following principle: *Two triangles are* **congruent**

if two angles and the side included between their vertices in one triangle are equal, respectively, to the corresponding parts of the other.

Exercise 5 shows how a surveyor may use this principle.

5. It is required to determine the distance from a point A on the shore line (Fig. 154) to a point C in the lake.

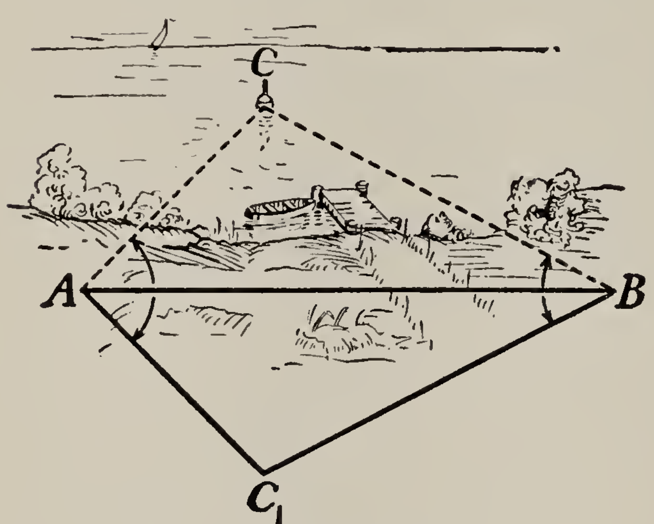


FIG. 154

A base line AB is laid off along the shore.

The transit is then placed at A and angle BAC is measured.

Line AC_1 is drawn, making angle BAC_1 = angle BAC.

Similarly, angle ABC_1 is laid off equal to angle ABC.

Show that triangle $ABC \cong$ triangle ABC_1.

Show that the length of AC may be found by measuring AC_1.

77. The nature and value of land surveying. References to surveying have been made repeatedly in the preceding pages. The art of surveying, *i.e.*, of measuring land, is probably as old as civilization. In Egypt, the science of surveying helped people to re-establish each year the boundaries of their land obliterated by the inundations of the Nile. The purpose of surveying is to establish certain boundaries of land and later to identify and locate these boundaries. The need of this is seen in our own history when the pioneer settlers followed Daniel Boone into the wilds of Kentucky and settled upon patches of land. They frequently marked their boundaries of land, or their *claims*, by chopping notches in trees. As time went on,

the crudeness of this method resulted in many disagreements concerning the ownership of particular patches and frequently in feuds.

On the other hand, when the Northwest Territory was opened to settlement about 1788 the Ohio Company of Associates, which purchased 1,000,000 acres of land, took great care to have the land properly subdivided and recorded and to have each settler's property clearly labeled and defined. For this purpose they

employed surveyors who traveled for miles through the wilderness, measuring the land and placing large stones for permanent landmarks, the location of each being entered in the records. Later by congressional ordinance this work was extended by dividing the land into townships, the townships into sections, and the sections into quarter-sections.

Accuracy in surveying is exceedingly important. In a small town near Chicago, the surveyor in laying out village blocks used a surveyor's chain for measuring which was slightly too long. Later when each block was divided into lots there always remained a small piece of land in every block which did not belong to any lot. Frequent quarrels arise even to this day between claimants of this odd piece in each block.

Can you give examples from your reading of disagreements over staked claims in primitive settlements?

Do you know of disputes arising from peculiarities in land surveying?

What surveyor's marks have you seen in your neighborhood?

What American president was a skilled surveyor?

Find out what is meant by the Mason-Dixon line.

78. Instruments used in surveying. We have seen that the surveyor measures angles with the transit.

For measuring *distances* he uses chains, tapes, and rods. The *chain* (Fig. 155) consists of 100 links made of heavy steel wire. At every tenth link there is a brass tag, and at each end there is a handle. Some chains are 66 ft. long, the length of each link being

7.92 inches. Others have each link 1 ft. long, making the length of the chain 100 feet. Chains are used when the territory is rough and extreme accuracy not essential.

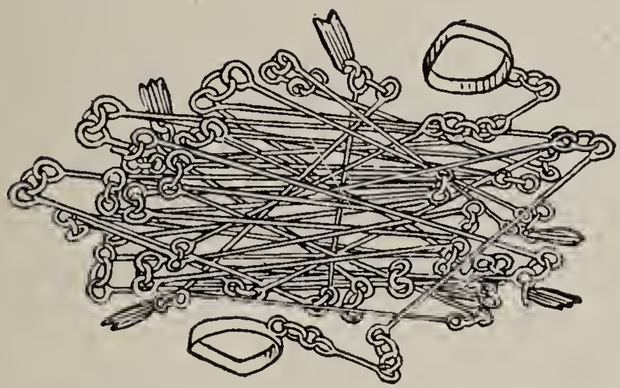


FIG. 155. SURVEYOR'S CHAIN

FIG. 156. STEEL TAPE

FIG. 157. CHAINING PINS

The *tape* (Fig. 156) is more convenient, less bulky, and more accurate than the chain. Tapes are usually 50 or 100 ft. long.

The *chaining pins* (Fig. 157) are used to mark points on the ground.

THE SCALE-DRAWING METHOD OF FINDING UNKNOWN DISTANCES

79. Scale drawing. We have seen that distances which cannot be measured directly, such as the width of a river, or the height of a chimney, may sometimes be found by laying off a triangle congruent with a triangle having the required distance as one side. This is the *congruent-triangle method of indirect measurement.* When it is impossible, or inconvenient, to lay off the triangle, other methods of finding the unknown distance are needed. The following example illustrates a method known as the *scale-drawing method.*

Let it be required to find how far it is from one corner of the classroom to the opposite corner.

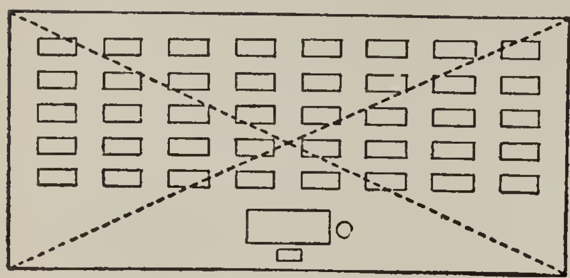


Fig. 158

Solution: Two consecutive sides of the room are measured and found to be 20 ft. and 30 ft. long, respectively (Fig. 158).

Let the side of a small square on squared paper represent 1 foot (Fig. 159).

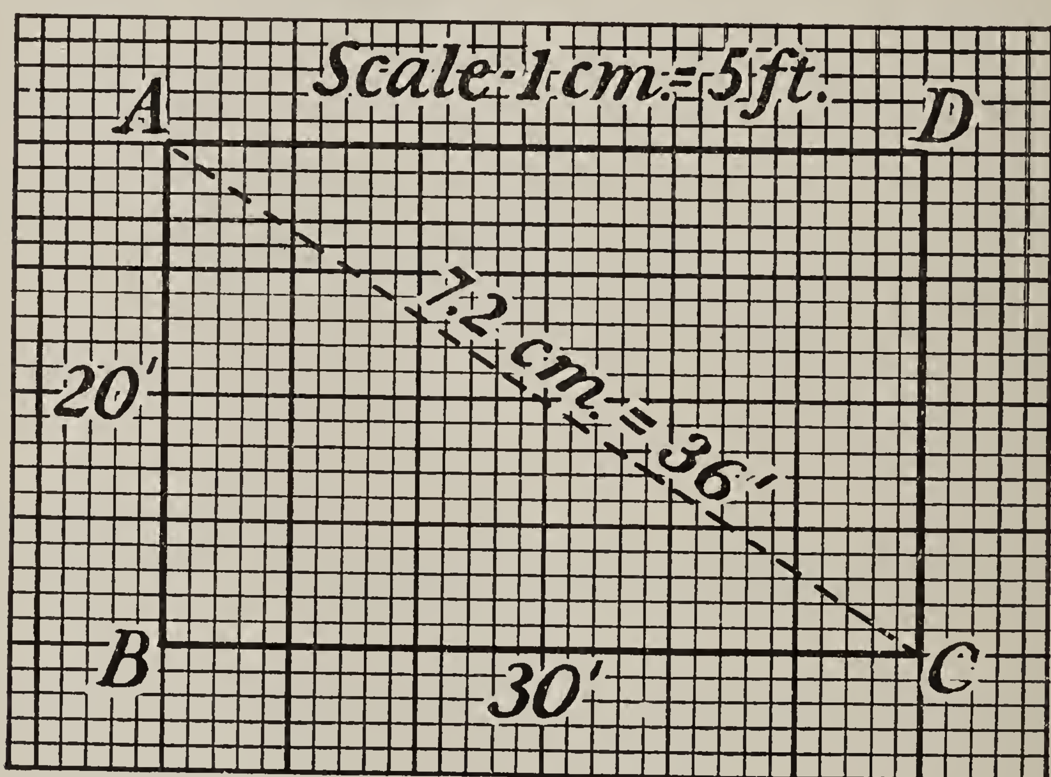


Fig. 159

Draw $BC = 6$ cm., to represent the 30 ft. side.
Draw $BA = 4$ cm., to represent the 20 ft. side.
Complete the rectangle $ABCD$.

Then $ABCD$ is a *scale drawing* of the classroom floor with dimensions proportionally smaller than the *actual* dimensions. Thus, a length in the drawing equal

to a centimeter represents a length of 5 ft. of the floor. The drawing is then said to be made to the *scale:* 1 cm. = 5 ft.

The distance from A to C in the drawing is found by measurement to be about 7.2 cm., the 2 being estimated and therefore doubtful. The actual distance is (7.2×5) ft., or 36 feet approximately.

Summary: The preceding method of finding the distance AC involves the following steps:

1. Lines and angles *related* to the required distance are measured. In the example above we measured AB, BC, and $\angle B$ (Fig. 159).

2. A convenient scale is selected, measured lines are drawn to scale on squared paper, and the measured angles are drawn where they are needed to complete the figure.

3. The line representing the *required* distance, as AC, is then drawn and measured.

4. The measurement obtained from the scale drawing is changed to the actual length.

Knowing how to draw to scale is important as it enables us to understand plans of land made by the surveyor, the maps used in geography, and the blue prints of the architect. The problem above shows that by measuring lines on the scale drawing we can determine the lengths of parts of the objects which the drawing represents, or the distances which cannot be measured directly. For this reason the *scale* must always be stated on the drawing.

80. Diagonal. A segment which joins two vertices of a polygon which do not lie in the same side is a **diagonal** of the polygon.

Exercises

1. Find the scale on a map of the state in which you live.

2. In the design of the hand mirror (Fig. 160) determine the *actual* lengths indicated by the dotted lines.

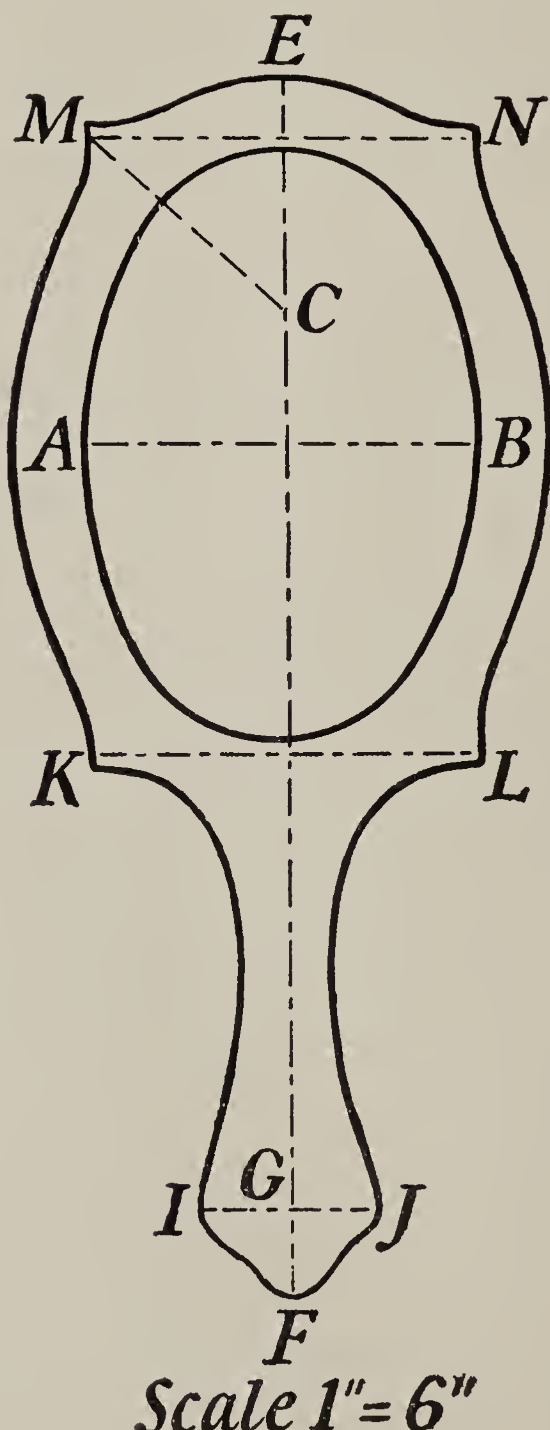


Fig. 160

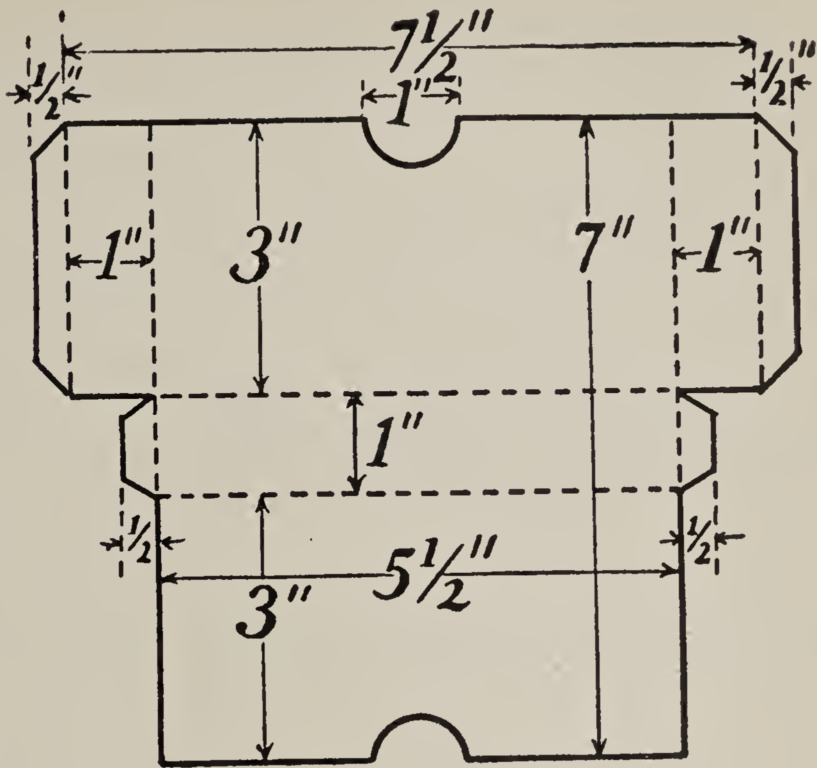


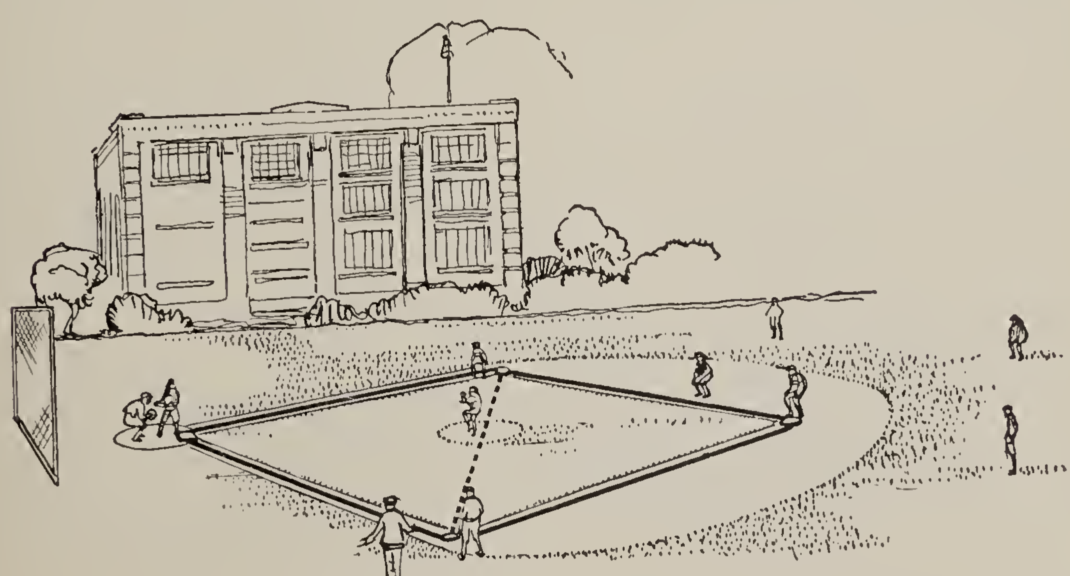
FIG. 161

3. Find the scale used in the diagram (Fig. 161).

4. Draw the design (Fig. 161) in actual size on a piece of heavy cardboard, cut along the solid lines, and then bend along the dotted lines. By joining the edges together by means of the flaps, a useful envelope case will be obtained. This may be mounted on a board $7\frac{1}{2}'' \times 3''$.

In working out the exercises below follow the directions given in §79.

5. A baseball diamond is of the form of a square whose side is 90 ft. long. Make a scale drawing of the diamond and find the direct distance of a throw from first to third base.



6. A man starting from a point P walks 60 yd. west and then 35 yd. north. What is his direct distance from P?

Suggestion: On a scale drawing to the right is *east,* and to the left is *west.*

7. In building a barn a carpenter aims to make the height of the roof (Fig. 162) equal to one-fourth of the width of the building. By means of a scale drawing find the angle between the rafter AC and the plate AB.

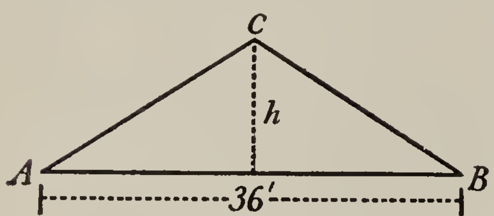


Fig. 162

8. Two automobiles leave a garage at the same time. One running at a rate of 20 miles an hour travels east for two hours, and then north for one hour. The other running at a rate of 25 miles an hour travels southwest for one hour and north for two hours. Find the distance between them at this time, using a scale drawing.

Suggestions: Make a rough sketch before attempting an accurate drawing. The required line is to be the *last* line drawn. Southwest means halfway between south and west (p. 89).

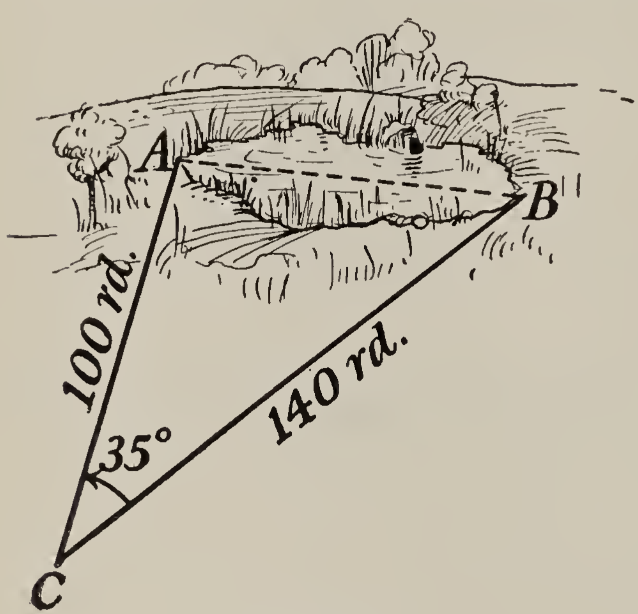


Fig. 163

9. Solve Exercise 1 (§76) by means of a scale drawing. The measured parts are given in Fig. 163.

Suggestion: Draw CA first.

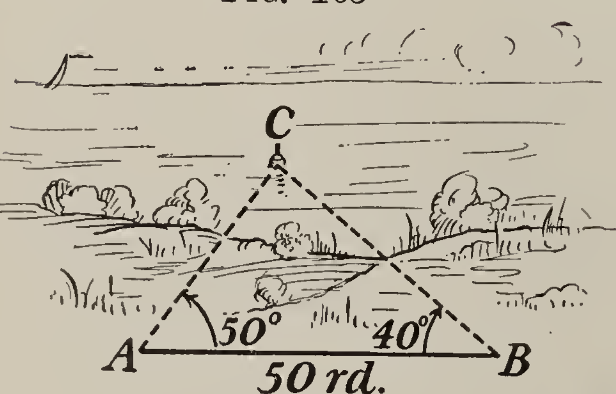


Fig. 164

10. Solve Exercise 5 (§76) by means of a scale drawing, the measured parts being shown in Fig. 164.

Suggestion: Draw AB first.

Check the accuracy of the drawing by finding the third angle.

11. A surveyor wishes to find the distance AB (Fig. 165) between two points, A and B, on opposite sides of a river. B is so located that it cannot be seen from A.

He first draws a straight line DC through A.

He then lays off a distance of 160 ft. from A to C.

Placing his transit at C he measures angle ACB and finds it to be 45°.

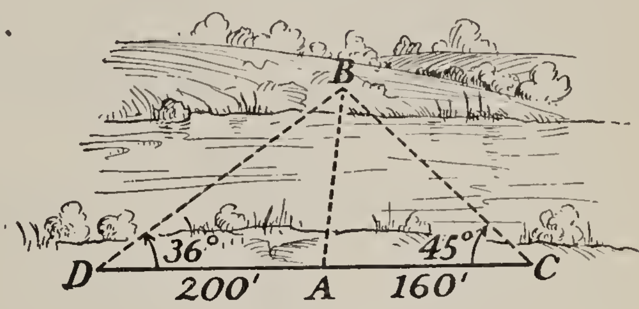


FIG. 165

Similarly, 200 ft. from A he locates D, and finds by measurement that angle $ADB = 36°$. Find the distance from A to B.

81. Angle of elevation. To find the height of a chimney AB (Fig. 166) a surveyor places his transit at a point C taken at a convenient distance from A.

He then points the telescope horizontally in the direction ED.

Turning the telescope through angle DEB, he points it to the top of the chimney in the direction EB. Angle DEB is called the **angle of elevation** of the point B from the point E.

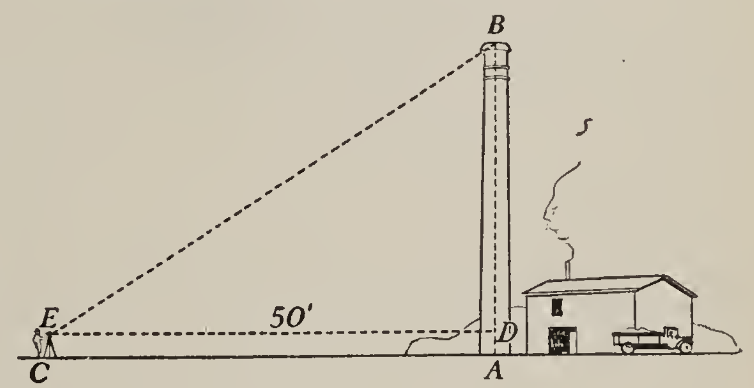


FIG. 166

EXERCISES

1. The telescope (Fig. 166) is 4 ft. above the ground and 50 ft. from A. AC is in horizontal position. Measure the angle of elevation, make a scale drawing of $\triangle EDB$, and from it find the length of DB. Find AB.

2. The angle of elevation of the top of a tree is 30° when observed at a point 40 ft. from the foot of the tree. How high is the tree above the horizontal line of sight?

To test the accuracy of the drawing use §66, Exercise 11, *i.e.*, measure the hypotenuse of the triangle and compare its length with the length of the side opposite to the 30° angle.

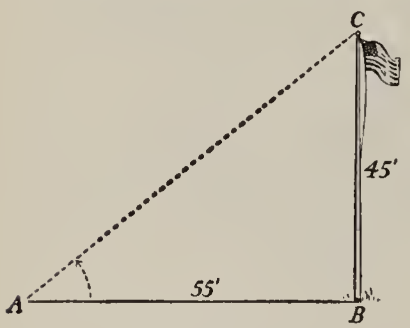


FIG. 167

3. A flagpole (Fig. 167) 45 ft. high casts a shadow 55 ft. long. Make a scale drawing and find $\angle CAB$. This is the angle of elevation of the sun.

4. When the angle of elevation of the sun is 30° a building casts a shadow 85 ft. long. Find the height of the building. Can you state a convenient way of checking the accuracy of your drawing?

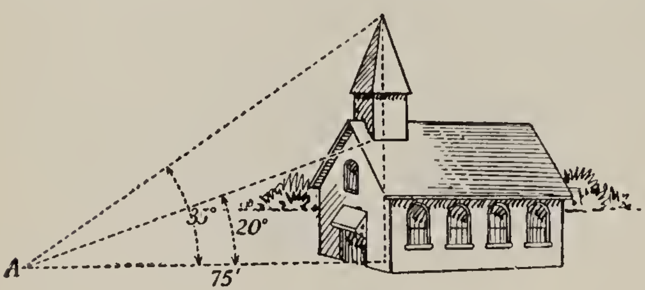


FIG. 168

5. On top of a building (Fig. 168) is a tower. At a point A, 75 ft. from the base of the building, the angle of elevation of the top of the tower is 35°, and the angle of elevation of the base of the tower is 20°. Find the height of the tower.

6. To determine the height of a tower AB (Fig. 169) a surveyor places his transit at a point C and finds the angle of elevation EC_1B to be equal to 68°.

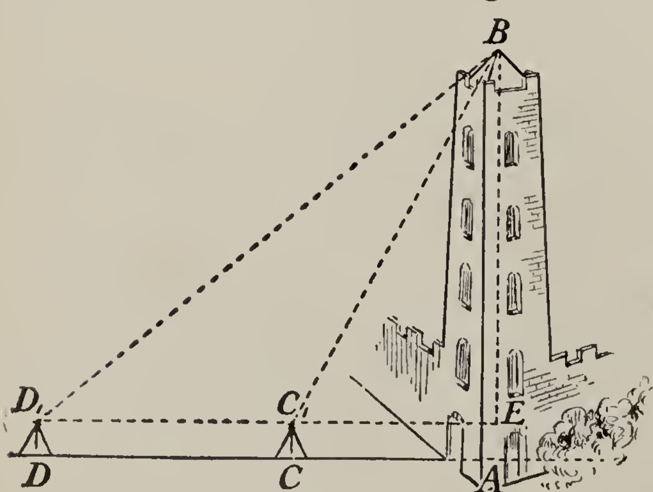


FIG. 169

He next places the transit at a point D in line with C and E and 60 ft. from C. He finds the angle of elevation ED_1B to be 35°. If in both cases the telescope was above the ground, find [illegible] length of AB.

82. Angle of depression. A transit is placed on top of a cliff A (Fig. 170) overlooking a river.

The telescope is first pointed *horizontally* in the direction AC.

It is then turned through angle CAB until it points

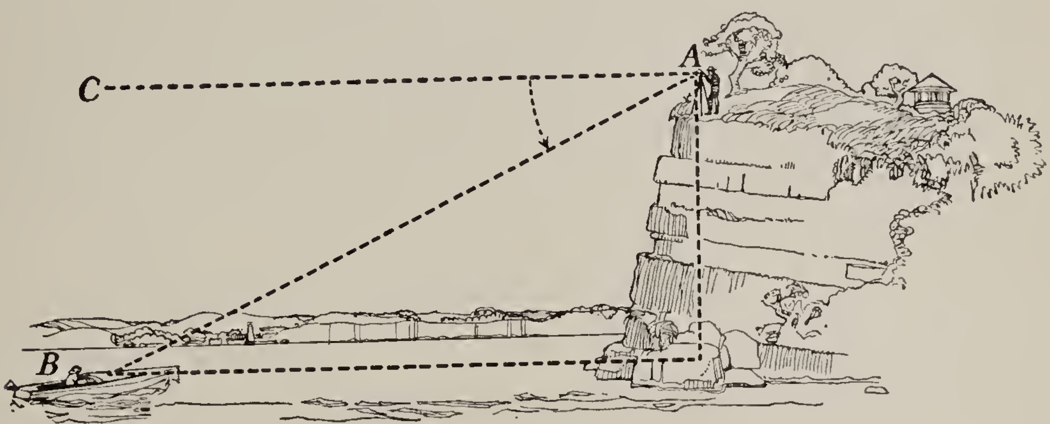


FIG. 170

to a passing boat B. The angle CAB is called the **angle of depression** of the boat from the point A.

Show that the *angle of depression* of B from the point A is the same as the *angle of elevation* of A from the point B.

EXERCISES

1. From the top of a lighthouse 100 ft. high (Fig. 171) the angle of depression of a boat is 50°. How far is the boat from the top of the lighthouse?

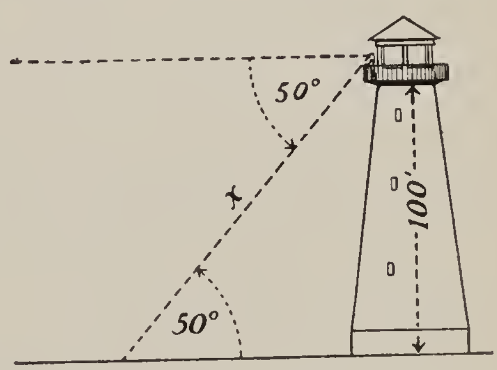


FIG. 171

2. An observation balloon B is anchored 2000 yd. above a point A. The balloonist observes the enemy at a point C and finds the angle of depression of C from B to be 62°. How far is it from A to C?

3. From the top of a vertical cliff 100 ft. high the angle of depression of a buoy is 30°. Find its distance from the bottom of the cliff.

The Similar-Triangle Method of Finding Unknown Distances

83. Advantages of the method. The use of the congruent-triangle method of finding distances is limited because it requires a triangle to be laid off which is congruent with the triangle containing the desired distance as a side. The scale-drawing method is an improvement because the required triangle is drawn to scale on *paper*, not actually on the ground. However, the errors introduced in making a drawing, and the time spent in attempting to attain a high degree of accuracy in drawing and measuring, offer serious objections to this method. The "method of similar triangles," which is the next to be studied, has the advantage that an *exact* drawing is not needed, a rough sketch being sufficient. Furthermore, the final result is not determined by measurement, but by solving an equation, which makes greater accuracy possible.

84. Similar triangles. On squared paper draw a triangle, as ABC (Fig. 172).

Draw $A_1 B_1$ not equal to AB.

On segment A_1B_1 construct triangle $A_1B_1C_1$ so that angle $A =$ angle A_1, and angle $B =$ angle B_1.

Tell, without measuring, how angle C should compare with angle C_1.

Measure angles C and C_1 to test the accuracy of the drawing.

How do the two triangles compare as to shape? As to size?

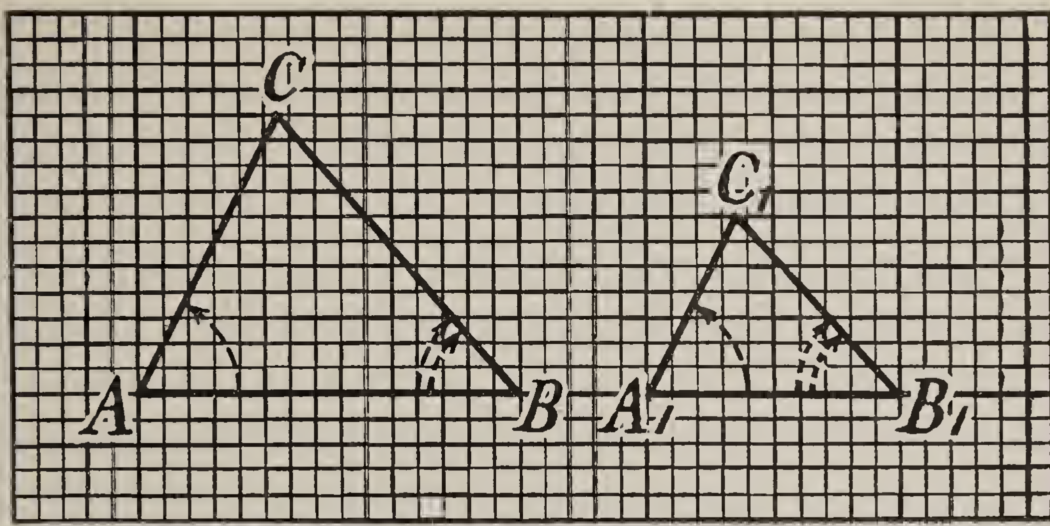


FIG. 172

Triangles having the same shape are *similar triangles.* Similar triangles do not have to be of the same *size.*

Using 2 cm. as a unit measure all the sides of the two triangles to two decimal places.

Compute to two decimal places the ratios

$$\frac{AC}{A_1C_1} = \frac{1.67}{1.23} = 1.36, \text{ the 6 being doubtful.}$$

$$\frac{BC}{B_1C_1} =$$

$$\frac{BA}{B_1A_1} =$$

Computation:

```
       1.36
1.23)1.67
     1 23
      440
      369
       71
```

If the measurements were accurate and the divisions exact, these ratios would be equal.

85. Similar polygons. The preceding exercise illustrates the following geometric facts:

1. *If the angles of one triangle are respectively equal to the angles of another, the two triangles are similar.*

2. *If the corresponding angles of two triangles are equal, the ratios of the corresponding sides are also equal.*

These two principles form the basis for the following definition of **similar polygons.**

3. *Two polygons are* **similar** *if the corresponding angles are equal and if the ratios of the corresponding sides are equal.*

Similarity has been shown in geometrical polygons. However, every figure drawn to scale is similar to the original. Maps and charts are similar in shape to the region which they represent. A photograph of a building is similar to the building whose picture is taken. Knowledge of similarity is of importance to those who expect to become architects and designers, and to the engineers and machinists who have to use the plans drawn by the draftsmen.

86. Symbol for similarity. The symbol for similarity is $\backsim$ (§75). Thus, the statement *triangle ABC is similar to triangle* $A_1B_1C_1$ may be written briefly,

$$\triangle ABC \backsim \triangle A_1B_1C_1$$

Many objects are of the same shape and therefore similar, *e.g.*, any two squares, two equilateral triangles, a scale drawing and the figure it represents, the map of a piece of land and the land itself.

87. The similar-triangle method. The following example illustrates the method of finding distances by means of similar triangles.

To measure the height, h, of a vertical pole AC (Fig. 173) a boy measures the length of the shadow

AB and finds it to be 76 ft. long. At the same time he finds that the shadow A_1B_1 of a 5 ft. vertical pole is 8 ft. long. What is the height of the first pole?

Solution: Each pole, its shadow, and the sun's ray passing over the top of the pole form a triangle.

The two triangles thus formed have two corresponding angles equal. For, angles A and A_1 are right angles, and angles B and B_1 are equal angles of elevation of the sun.

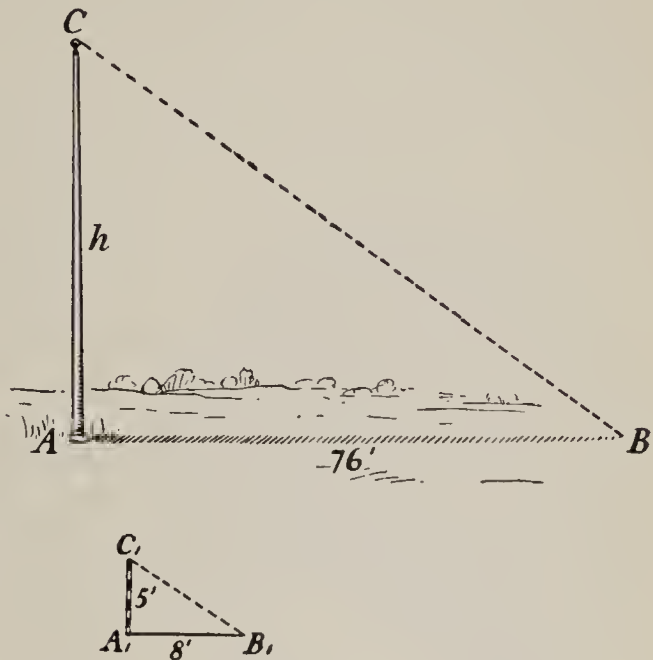


Fig. 173

Hence $\triangle ABC \backsim \triangle A_1B_1C_1$. Why?

It follows that the ratios of the corresponding sides are equal, for example that

$$\frac{h}{5}=\frac{76}{8}.$$

We shall now learn how to solve this equation.

Multiplying both sides of the equation by 5, we have

$$\frac{\cancel{5}\times h}{\cancel{5}}=\frac{5\times\overset{19}{\cancel{76}}}{\underset{2}{\cancel{8}}}$$

By reducing the fractions it follows that

$$h=\frac{5\times 19}{2}=47\tfrac{1}{2}.$$

Hence the pole is about $47\frac{1}{2}$ ft. high.

The Greek mathematician Thales of Miletus (600 B.C.) is said to have used this method, known as the *shadow method*, to determine the height of the pyramids.

The adjoined picture of Thales expresses an artist's idea of the face of this famous ancient mathematician. You will find it interesting to know something about his life and work. He is known as the founder of the earliest Greek school of mathematics, but probably did not know as many mathematical facts as you do. In books on the history of mathematics you will find interesting stories about him.

THALES

88. Multiplication axiom. Equations of the type $\frac{h}{5}=\frac{76}{8}$ (§87) are met frequently in the solution of problems. Hence, we should understand how to solve them. In solving the equation, it was assumed that both members may be multiplied by the same number without destroying the equality. This principle of mathematics, known as the **multiplication axiom,** is usually stated as follows: *if equal numbers are multiplied by the same number the products are equal.* Since equations of the type above occur in the similar-triangle method, Exercises 1 to 9 on page 131 are designed to give practice in the use of the multiplication axiom in the solution of equations.

Exercises

Solve the following equations:

1. $\frac{x}{18}=\frac{4}{9}$.

Solution: $\frac{x}{18}=\frac{4}{9}$.

$$\frac{\cancel{18}x}{\cancel{18}}=\frac{4\times\overset{2}{\cancel{18}}}{\cancel{9}}$$, by multiplying both members by 18. (Multiplication axiom)

$x=8$, by reducing the fractions.

Check.

$$\frac{8}{18}=\frac{4}{9}$$

$$\frac{4}{9}=\frac{4}{9}.$$

2. $\frac{w}{25}=\frac{8}{15}$.

3. $\frac{l}{7}=\frac{3}{14}$.

4. $\frac{y}{10}=\frac{19}{6}$.

5. $\frac{a}{9}=\frac{5}{21}$.

6. $\frac{2}{3}=\frac{x}{24}$.

7. $\frac{m}{56}=\frac{75}{44}$.

8. $\frac{z}{8}=\frac{4}{12}$.

9. $\frac{7}{12}=\frac{x}{10}$.

Solve the following problems:

10. A boy scout, wishing to find the height of a pole (Fig. 174), holds a stick $\frac{1}{2}$ ft. long in vertical position, far enough from his eye so that it just covers the pole. If he stands 150 ft. from the pole, and if the stick is held 2 ft. from his eye, find the height of the pole.

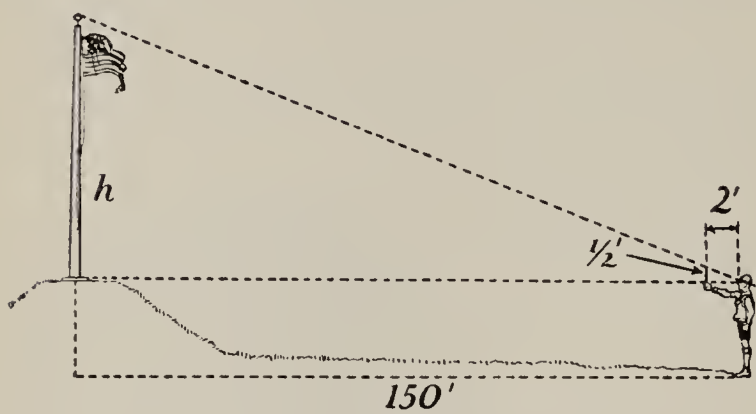


Fig. 174

Suggestions: Make a sketch of Fig. 174. Locate two similar triangles in the drawing, and equate the ratios of the corresponding sides. Then solve the equation.

11. How high is a tree which casts a shadow 43 ft. long, if at the same time a 7 ft. vertical post casts a shadow 9 ft. long?

Follow the suggestions given in Exercise 10 and find the height by solving an equation.

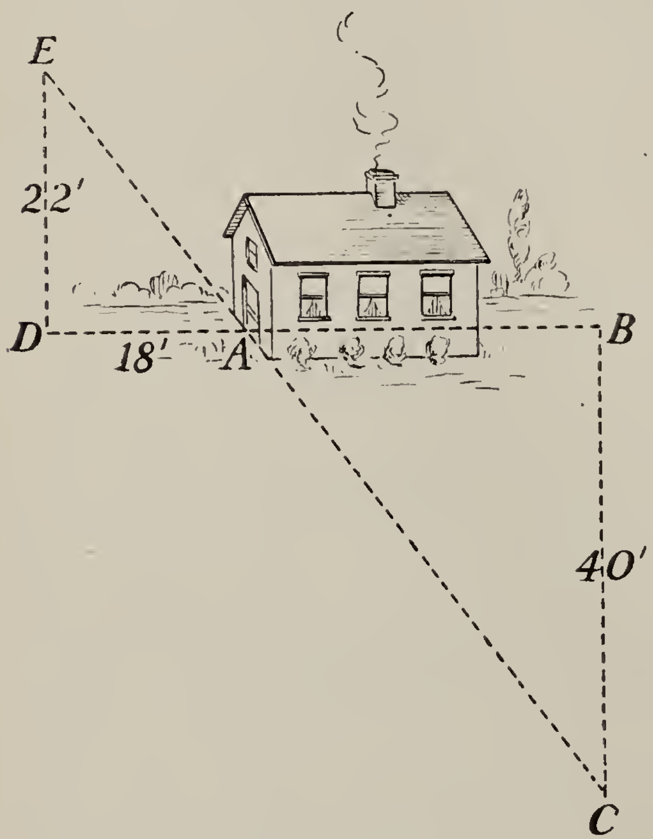


Fig. 175

12. To find the distance AB (Fig. 175) through a building, BC and DE were drawn at right angles to the line DAB. The following measurements were then made

$BC = 40$ ft.

$DE = 22$ ft.

$DA = 18$ ft.

What is the length of AB?

13. The sides of a triangle are 5 ft., 7 ft., and 8 feet. The longest side of a similar triangle is 15 ft. Find the other two sides.

14. The floors of two rooms in a new building are to be similar. The dimensions of one room are to be 15 ft. and 20 feet. The other room is to be 18 ft. long. Find the width.

15. The sides of a triangle are 2.9″, 5.2″, 6.3″. The shortest side of a similar triangle is 2.4″. Find the other two sides to two figures.

16. Find the height of your school building using the *shadow method* described in §87.

89. Proportion. A road bed AC (Fig. 176) rises to a height of 38 ft. for a horizontal distance AB of 110 feet. Let it be required to find how many feet the bed rises for a horizontal distance of 32 ft.

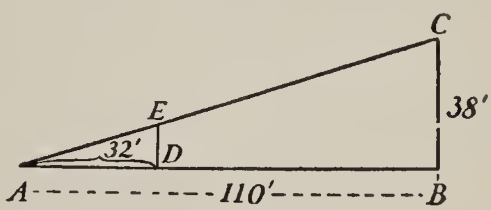


Fig. 176

This problem will be worked by two methods.

a. Solution by the scale-drawing method. On squared paper draw to a convenient scale $AB = 110'$, $AD = 32'$, and $BC = 38'$. Draw $DE \perp AB$. Measure DE. This is the required length.

To check the accuracy of the result compute the ratios $\frac{DE}{38}$ and $\frac{32}{110}$ to three figures. Each of these ratios expresses the relative size of two distances. They should agree to two figures if careful work is done.

If two ratios, as $\frac{DE}{38}$ and $\frac{32}{110}$, are equal the statement $\frac{DE}{38} = \frac{32}{110}$ is called a *proportion.*

b. Solution by the similar-triangle method. Show that triangles ABC and ADE (Fig. 176) are similar. Then it

follows that the ratios of the corresponding sides are equal, *i.e.*, $\frac{DE}{38}=\frac{32}{110}$.

Solve this equation to determine DE.

An equation of two equal ratios is a **proportion**.

Thus, $\frac{4}{6}=\frac{8}{12}$ is a proportion. Why?

Form three proportions.

Is the statement $\frac{6}{10}=\frac{3}{4}$ a proportion?

A proportion, as $\frac{2}{3}=\frac{20}{30}$, is read "2 over 3 is equal to 20 over 30," or "2 is to 3 as 20 is to 30."

Read the proportions

$$\frac{4}{16}=\frac{1}{4};\ \frac{1}{2}=\frac{8}{16};\ \frac{9}{10}=\frac{18}{20};\ \frac{a}{b}=\frac{c}{d}.$$

The equation $\frac{DE}{38}=\frac{32}{110}$ expresses a relation between horizontal distances and the rise of a road. It means that the ratio of one horizontal distance to another is equal to the ratio of the rise corresponding to the first to the rise corresponding to the second horizontal distance.

This type of relationship is found frequently. The following are examples in which it occurs.

1. A train moving at uniform rate travels 280 miles in 8 hours. How far does it travel in 14 hours?

Here we have the ratio of the two distances equal to the ratio of the corresponding number of hours, *i.e.*,

$$\frac{280}{x}=\frac{8}{14}.$$

2. An alloy of silver and copper weighing 90 oz. contains 6 oz. of copper. How much copper is there in 30 ounces?

Show that $\frac{90}{30} = \frac{6}{x}$.

3. If a sum of \$935 yields an annual income of \$46.75, what will be the income on \$500?

Show that $\frac{935}{500} = \frac{46.75}{x}$.

90. Historical note. We have seen that if 3 of the 4 terms of a proportion are known, as in $\frac{x}{5} = \frac{76}{8}$, the fourth can be found by solving the equation for x. Rules for finding the fourth number of a proportion when three are known have been given considerable emphasis in the study of mathematics because of their great usefulness in the solution of many types of problems. Since three numbers are required in the operation, the method of finding the fourth became known as the *Rule of Three*. It has also been named the *Golden Rule*. "*The Rule of Three* is the chiefest and most profitable and the most excellent rule of all arithmetike for which cause it is said philosophers did name it the golden rule."—Humphrey Baker (1562).

The theory of proportion is as old as Plato's time (427–347 B.C.). There has been considerable diversity among mathematicians as to the notation used in writing proportions.* The question: if 2 apples cost 8

* See Cajori's *History of Elementary Mathematics*, pp. 193–203.

cents, what will 7 cost? in the notation of the Italian writer Tartaglia (1506–1557) would be stated thus:

If apples 2 || cost cents 8 || what be the cost of apples 7?

The older English arithmeticians write this as follows:

Apples   Cents
2 ———— 8
7   28

In the seventeenth century the following was customary:

Apples   Cents   Apples
2 ———— 8 ———— 7

Oughtred (1574–1660) wrote it

2 : 8 :: 7

Thomas Dilworth (1784) used the notation

2 ·· 8 :: 7 ·· 28.

The notation of Leibnitz (1646–1716)

$2 : 8 = 7 : 28.$

was brought into use in the United States during the first quarter of the nineteenth century. It is read: 2 is to 8 as 7 is to 28.

The rule of three has played an important rôle in problem solving in England, America, and Germany, especially in commercial circles.

91. Extremes and means. The *first* and *last* terms of a proportion, as $a : b = c : d$, are called the **extremes**; the *second* and *third*, the **means.** Likewise, in the proportion $\frac{a}{b} = \frac{c}{d}$ the terms b and c are the means,

and a and d are the extremes. Name the means and the extremes in the following proportions: $\frac{1}{2}=\frac{4}{8}$; $\frac{6}{5}=\frac{18}{15}$; $\frac{m}{n}=\frac{x}{y}$.

92. Fundamental property of proportion. Write a proportion, as $\frac{3}{7}=\frac{6}{14}$. Find the product of the means; the product of the extremes. How do these products compare? Form another proportion, and compare the products of the means with the product of the extremes.

The preceding examples illustrate the principle that *in a proportion the product of the means is equal to the product of the extremes.*

This principle suggests a convenient method of solving a proportion for the unknown number. This method is to be used in the exercises below.

Exercises

Solve the following problems by means of proportions:

1. A boat runs 30 mi. in 3 hours. How many miles will it run in 8 hours?

Solution: The ratio of the time of 3 hours to the time of 8 hours is $\frac{3}{8}$.

Denoting the required distance by x, the ratio of the corresponding distances is $\frac{30}{x}$.

Since these ratios are the same, we have

$$\frac{3}{8}=\frac{30}{x}.$$

Since the product of the extremes, $3x$, is equal to the product of the means, 8×30, we have

$$3x=30\times 8,$$

$$\therefore\ x=\frac{\overset{10}{\cancel{30}}\times 8}{\cancel{3}}$$

$$\text{or}\quad x=80.$$

2. If 12 acres of land yield 440 bushels of corn, at the same rate of yield how many acres would yield 200 bushels?

Suggestion: As in Exercise 1, state the proportion and then solve the equation.

3. A farm valued at \$11,400 is taxed for \$76.38. At the same rate what would be the tax on a farm valued at \$14,250?

4. A boy pays 90 cents for 2 doz. oranges. What is the price of 20 oranges?

5. Five bars of soap are sold for 35 cents. At the same rate, find the price of 8 bars.

6. If the simple interest on a sum of money for 6 years is \$200, what will be the interest for 10 years?

7. The dimensions of a rectangle are 9 inches and 5 inches. Find the width of a similar rectangle 12 inches long.

Suggestion: Make a sketch before writing the equation.

8. The shadows of a pole and a 5-foot rod are respectively 80 feet and 7 feet. Make a sketch and then find the height of the pole by means of an equation.

9. If a bushel of shelled corn weighs 56 lb., how many ounces does a pint weigh?

10. The food parts of beef are protein, fat, and water. In 5 lb. of sirloin steak there are 13 oz. protein, 14 oz. fat, and 42 oz. of water, the remainder being waste material. How many ounces of protein, fat, and water are there in 3 lb. of sirloin steak?

11. If $\frac{7}{8}$ yd. of lace cost 63 cents, how many yards can be purchased for $3.50?

12. If 3 yd. of ribbon cost $2.70, what is the price of $1\frac{3}{4}$ yards?

13. If an automobile runs 16 mi. on a gallon of gasoline, how much gasoline will be consumed on a 350-mile trip?

14. If a stenographer writes 475 words in 3 minutes, how long will it take her to write 2000 words?

15. If a $64.80 tax is paid on property assessed at $2575, what tax should be paid on property assessed at $6000?

16. If 230 lb. of milk produce 8.3 lb. of butter fat, how many pounds of milk will be required to produce 35 lb. of butter fat?

17. If 86 lb. of metal make 15 castings, how much metal will be required to make 10 similar castings?

18. If 18 yd. of silk cost $48.50, what will 35 yd. cost?

The Right-Triangle Method of Finding Unknown Distances

93. Advantages of this method. Several methods have been used to determine unknown distances by indirect measurement. Each method is an improvement over the preceding methods. As we continue the study of mathematics, we also continue to improve our methods. In the method explained below the number of measurements required to solve the problem is

reduced to a minimum. This also reduces the number of errors. The result does not depend on the accuracy of a drawing. For this reason it is generally used in practical work, such as surveying, where accuracy is important. It is called the *right-triangle method.* The method is made clear in §§94–96.

94. Similar right triangles. On squared paper draw a right triangle (Fig. 177) having one acute angle equal to 30°. Measure to two decimal places the side a, *opposite* the 30° angle, and the side b, *adjacent* to the 30° angle. Find the ratio $\frac{a}{b}$ by dividing a by b to two decimal places.

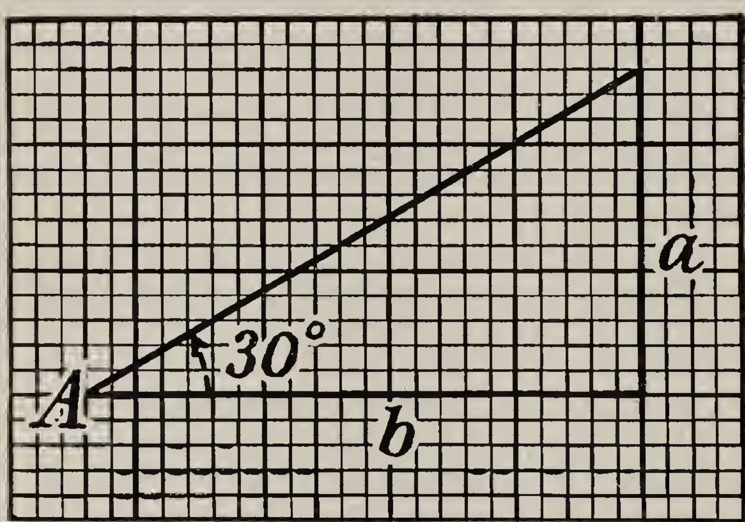


Fig. 177

The results found by the pupils in the class should agree. For according to §84 all these triangles, if constructed *exactly*, are similar to each other. They are therefore really the same triangle drawn to different scales.

95. Table of tangents. We have seen (§94) that for all right triangles having one acute angle equal to 30°, the ratio of the side *opposite* the 30° angle, to the side *adjacent* to the 30° angle is the *same.*

We may now draw other right triangles with acute angles of various sizes and make a table which states

for each acute angle the ratio of the side *opposite* to the side *adjacent*. Such a table is called a **table of tangents,** and the ratios are called **tangent ratios.** The

TABLE OF TANGENTS OF ANGLES FROM 0° TO 89°

Angle	*Tangent*	*Angle*	*Tangent*	*Angle*	*Tangent*
0	.000	30	.577	60	1.732
1	.017	31	.601	61	1.804
2	.035	32	.625	62	1.881
3	.052	33	.649	63	1.963
4	.070	34	.675	64	2.050
5	.087	35	.700	65	2.145
6	.105	36	.727	66	2.246
7	.123	37	.754	67	2.356
8	.141	38	.781	68	2.475
9	.158	39	.810	69	2.605
10	.176	40	.839	70	2.747
11	.194	41	.869	71	2.904
12	.213	42	.900	72	3.078
13	.231	43	.933	73	3.271
14	.249	44	.966	74	3.487
15	.268	45	1.000	75	3.732
16	.287	46	1.036	76	4.011
17	.306	47	1.072	77	4.331
18	.325	48	1.111	78	4.705
19	.344	49	1.150	79	5.145
20	.364	50	1.192	80	5.671
21	.384	51	1.235	81	6.314
22	.404	52	1.280	82	7.115
23	.424	53	1.327	83	8.144
24	.445	54	1.376	84	9.514
25	.466	55	1.428	85	11.430
26	.488	56	1.483	86	14.301
27	.510	57	1.540	87	19.081
28	.532	58	1.600	88	28.636
29	.554	59	1.664	89	57.290

table on page 141 gives the tangent ratios* for acute angles from 0° to 89°. Thus for an angle of 30° we find the tangent ratio to be .577. Briefly, we say *the tangent of 30° is .58,* which may be written

$$\tan 30° = .58$$

EXERCISES

1. From the table of tangents find the tangent ratios of the following angles: 10°; 22°; 45°; 67°; 82°.

In each case state your result in the form of an equation, as $\tan 10° = .176$.

2. Find the angles corresponding to the following tangent ratios: .141; .306; .601; 1.73; 6.31; $\frac{5}{4}$; $\frac{7}{8}$. State results in the form of equations.

96. How to use the right-triangle method. Exercise 1, below, shows how to find unknown distances by measuring two parts of a right triangle, and using the table of tangents. This is the **right-triangle method.**

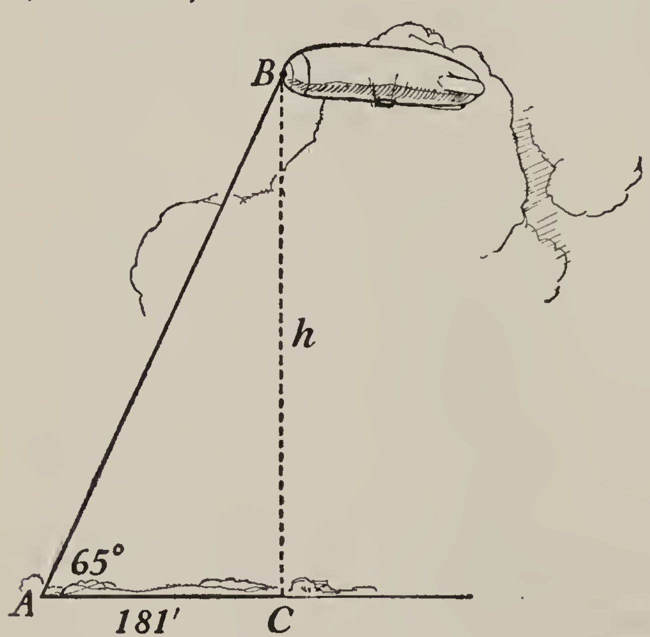


FIG. 178

EXERCISES

1. An observation balloon B (Fig. 178) fastened by a cable AB at A, is directly over a point C which is about 181 ft. from A. The angle of elevation of B at A is 65° approximately. How high is the balloon?

* There are other ratios, *e. g.*, the opposite side to the hypotenuse, and the adjacent side to the hypotenuse. The table of tangents is sufficient for the solution of the problems which follow.

Solution: $\dfrac{h}{181} = \tan 65°$. Why?

From the table, $\tan 65° = 2.145$.

Hence, $\dfrac{h}{181} = 2.145$.

Multiplying both members of the equation by 181,

$$h = (181) \times (2.145) = 388.245.$$

Since in the product (181) (2.145) the 5 in the 2.145 and the 1 in 181 are uncertain, the first partial product 2145 is uncertain. Furthermore, in the other partial products, the 8×5 and 1×5 are uncertain. Hence, in the sum of the partial products the last four figures are uncertain.

Computation:

$$\begin{array}{rl}
\underline{2.14\dot{5} \times 18\dot{1}}, & \\
\dot{2}\dot{1}\dot{4}\dot{5} & = \text{first partial product,} \\
1716\dot{0} & = \text{second partial product} \\
\underline{214\dot{5}} & = \text{third partial product} \\
38\dot{8}.\dot{2}\dot{4}\dot{5} & = \text{product.}
\end{array}$$

All the doubtful numbers in the computation above have been marked with dots placed over the number. Since the second 8 in 388 is doubtful, the figures following are meaningless, and should be dropped.

$\therefore$ The balloon is about 388 ft. high.

2. Make a summary of the steps in the solution of Exercise 1.

3. The rope of a flagpole is stretched so that it touches the ground at a point 18 ft. from the foot of the pole. The rope makes an angle of 70° with the ground. Find the approximate height of the pole, writing your computation as shown in Exercise 1.

4. A pole 22 ft. high casts a shadow 16 ft. long. Find the angle of elevation of the sun.

Solution: Let x be the required angle.

$$\tan x = \frac{22}{16}.$$

We may assume that in the measurement of the pole and the shadow the last figures in 22 and 16 were approximate. The division shows that in the quotient the first figure to the right of the decimal is doubtful and the second meaningless.

Hence $\tan x = 1.4$, and from the table

$$x = 54° \text{ approximately.}$$

Computation:

$$\begin{array}{r} 1.37 \\ \hline 1\dot{6})2\dot{2} \quad \\ 16 \quad \\ \hline 6.0 \\ 4.8 \\ \hline 120 \\ 112 \\ \hline \end{array}$$

5. A vertical pole 9 ft. long casts a shadow, on level ground, 11 ft. long. Find the angle of elevation of the sun.

Fig. 179

6. What is the angle of elevation of a road which rises $2\frac{1}{2}$ ft. in a horizontal distance of 25 feet (Fig. 179)?

7. In the triangle (Fig. 180) find the approximate values of angle x for the following values of a and b.

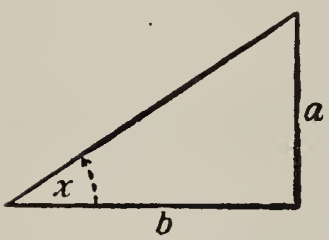


Fig. 180

a	231	18.4	1.57
b	172	15.3	1.83

8. Find the value of a (Fig. 180) corresponding to the value of x and b given in the following table:

b	165	28.3	204
x	20°	42°	35°

97. What every pupil should know and be able to do. Having studied Chapter V every pupil should be able to do the following:

1. To find unknown distances by the methods taught in the chapter.

2. To solve for the unknown equations of the type $\frac{x}{3}=\frac{16}{5}$.

3. To determine the degree of accuracy that can be obtained in the product of two decimal fractions in which the last figure to the right is doubtful.

4. To use compass, protractor, and squared paper in drawing triangles and angles, and measuring segments.

5. To solve some simple verbal problems by means of proportions.

The following principles should be known:

1. *Two triangles are congruent, if two sides and the included angle of one are equal to two sides and the included angle of the other.*

2. *Two triangles are congruent if two angles and the side included between their vertices in one triangle are equal respectively to the corresponding parts of the other.*

3. *If the angles of one triangle are equal to the angles of another, the triangles are similar.*

4. *If the corresponding angles of two triangles are equal, the triangles are similar, and the ratios of the corresponding sides are equal.*

5. *In a proportion the product of the means is equal to the product of the extremes.*

6. *If equal numbers are multiplied by the same, or equal, numbers the products are equal (multiplication axiom).*

7. The pupil should be familiar with the meaning of the following terms: congruent polygons, scale drawing, angle of elevation, angle of depression, similar polygons, proportion, tangent ratio.

98. Typical problems and exercises. The pupil should be able to solve the following problems and exercises:

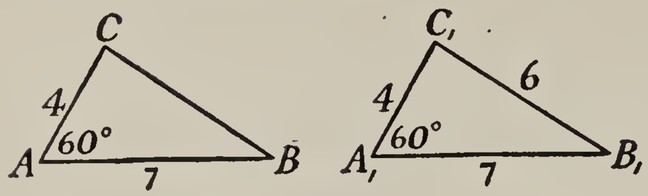


FIG. 181

1. In triangle ABC (Fig. 181) find the length of BC. Give reasons for your answer.

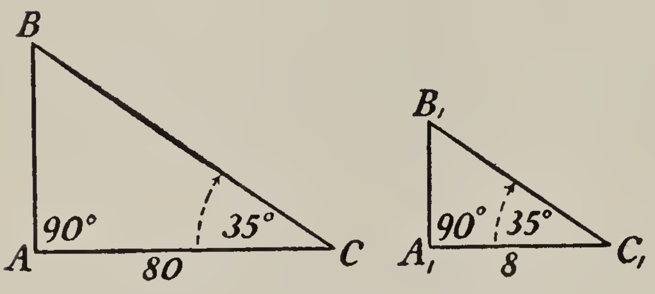


FIG. 182

2. In triangles ABC and $A_1B_1C_1$ (Fig. 182) find the lengths of AB and A_1B_1.

3. Draw a triangle. Draw a second triangle congruent with the first. Draw a third triangle similar to the first.

4. Find the distance AB (Fig. 183) by means of a scale drawing.

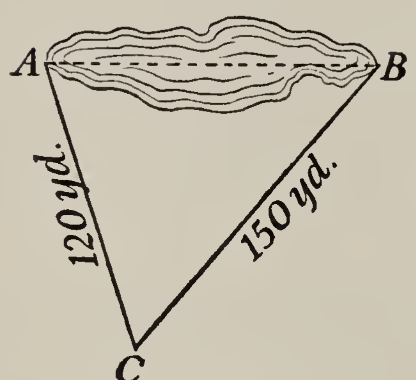


FIG. 183

5. Find the height of a building the shadow of which is 90 ft. long when the angle of elevation of the sun is 42 degrees. Use first the scale-drawing method; then the right-triangle method, tan 42 degrees being .900.

6. The length of the shadow of a flagpole is 72 ft., when the shadow of a 6 ft. vertical rod is 10 feet. Find the height by the similar-triangle method.

Solve for x:

7. $\frac{x}{3}=16$

8. $\frac{x}{10}=\frac{3}{5}$

9. $\frac{86}{x}=\frac{5}{8}$

10. $\frac{3.5}{6.3}=\frac{x}{.75}$

11. By means of a proportion, find the tax on a property assessed at $18,000, if a tax of $72.40 is paid on property assessed at $3280.

12. Determine to three figures the ratio $\frac{8.72}{9.36}$.

13. Write a paper on one of the following topics:

a. Uses of the triangle in ornamental work; in construction; in designing; in surveying.

b. Indirect measurement.

c. Land surveying.

d. Proportion.

e. Various methods of finding unknown distances.

f. Importance of congruence, similarity.

g. The life and work of Thales.

West Front, Westminster Abbey, Showing Gothic Architecture
of Great Britain

CHAPTER VI

THE CIRCLE

How the Circle Is Used

99. Meaning and uses of the circle. In the previous chapters, we have been studying figures formed by straight lines. We shall now take up the study of a well-known curved line, the circle. The circle is found so frequently in everyday life that everyone should know how to draw and use this curve.

The earliest nations showed great skill in the use of the circle in designs with which they beautified their temples and other important buildings. At a very early date men learned how to make use of the wheel for moving objects, having recognized that it required

Fig. 184. Design and Church Window Showing the Use of the Circle

only a forward movement and no lifting as in carrying.

A similar use of the circle has found its widest application in modern machinery, as in watches, lathes, sewing machines, and automobiles. We find it on tanks, boilers, pillars, doilies, phonograph records, baskets, and numerous other objects. It is used in designs and many other types of ornamental work.

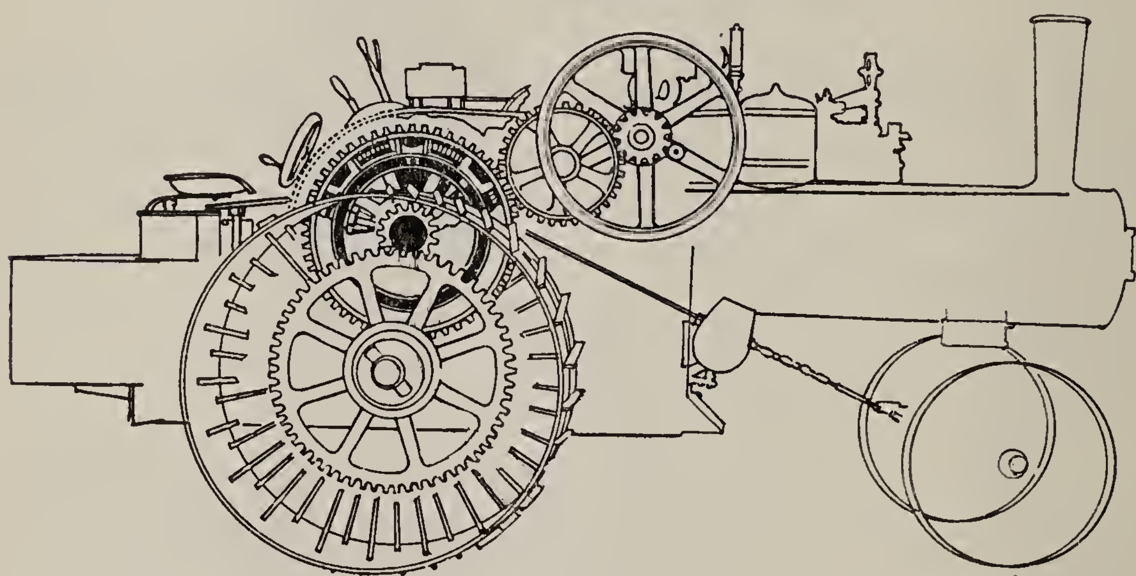


FIG. 185. THE USE OF CIRCLES IN A STEAM ENGINE

Note the circles and circular parts in the steam engine (Fig. 185) which is used to supply power for farms.

Gas and electric-light companies use the circle in measuring devices, such as gas and electric-light meters.

In geography we learn how circles determine the position of places on the earth's surface (Fig. 215). In astronomy stars are located and the exact time is determined by means of circles.

In geometry, drawings, or *constructions*, are based on the properties of the circle.

The circle is also used to represent numerical facts in a graphical way (Figs. 226 to 230).

It is the purpose of this chapter to acquaint you with some of these uses of the circle and to give you a clear understanding of the circle itself.

Drawing various designs and making the fundamental geometric constructions will help you develop skill in the use of the compass.

We shall see that the problem of finding lengths of circular lines leads to valuable experiences in solving equations, working with formulas, and making arithmetical computations.

100. How to draw a circle. Cut a piece of cardboard, as AB (Fig. 186), making it about 2 inches long and $\frac{1}{2}$ inch wide. With a pin, prick a hole near each end, at A and B.

Place the strip AB on a sheet of paper and fasten the end at A by means of a pin stuck through the hole.

Through the hole at B insert the point of a sharp pencil, and turn the strip about point A. The point of the pencil then traces a *curved line*, as BC. When a complete turn is made the curve *closes*. The closed curved line is called a *circle.*

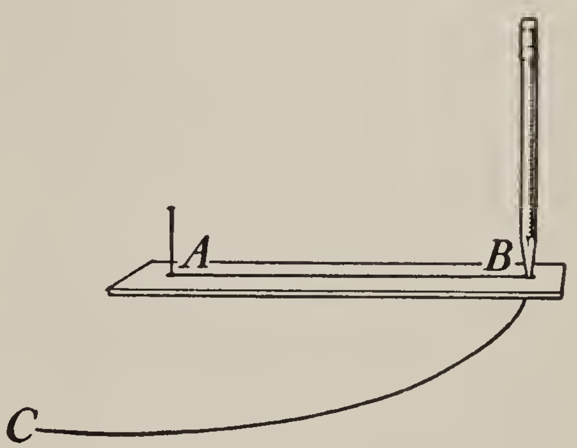


Fig. 186

All points on this line have the *same distance* from A. Thus, a **circle** is defined as *a closed curved line all points of which are equally distant from a fixed point.*

The fixed point is the **center** of the circle.

A line segment drawn from the center to a point on the circle is a **radius** (Fig. 187). A line segment drawn through the center and terminated by the circle is a **diameter** (Fig. 187).

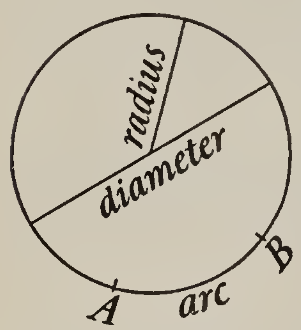


Fig. 187

A portion of a circle, as AB (Fig. 187), is an **arc**.

101. Notation for circles and arcs. The symbol for *circle* is ⊙. The symbol for *circles* is Ⓢ. Thus, ⊙A means: *a circle whose center is A.*

The symbol for *arc* is ⌒. "Arc AB" is written $\overset{\frown}{AB}$.

The Use of the Circle in Designs

102. How to draw a circle with the compass. A circle is more easily drawn with the compass (Fig. 188) than with a piece of cardboard, as shown in §100. When drawing a circle, hold the compass with one hand, between the thumb and forefinger, and in turning the compass press down lightly on the sharp point.

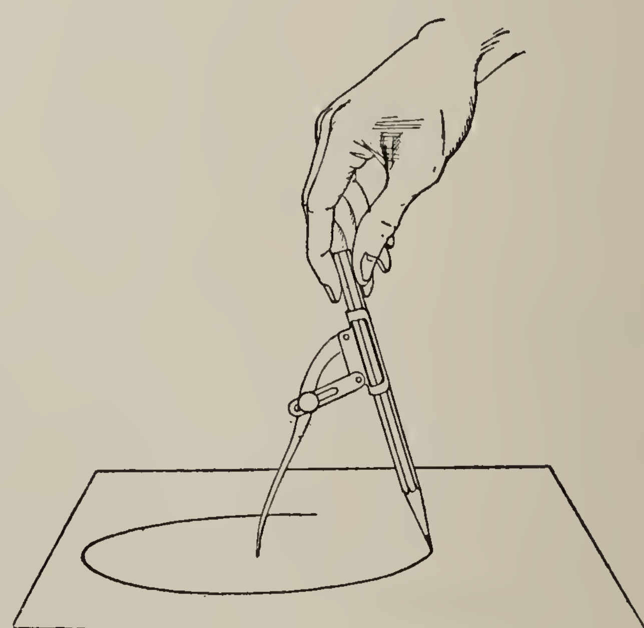


Fig. 188

The exercises found on the following pages give practice in drawing circles:

Exercises

1. Draw several circles on paper. With the blackboard compass draw circles on the blackboard.

2. Name several objects whose boundary lines are circles.

3. On a sheet of paper mark a fixed point A. Locate all the points on the paper which are 5 cm. from A. What seems to be the location of these points?

4. Draw a circle whose radius is 3.5 centimeters.

5. Draw a segment AB, 6 cm. long. Using A as center and a radius equal to 2 cm., draw a circle. Using B as center and a radius equal to 4 cm., draw a second circle.

6. Draw a segment AB, 8 cm. long. Then construct the design shown in Fig. 189.

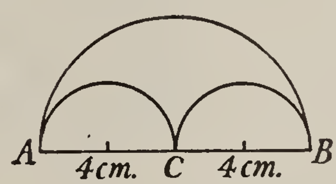


Fig. 189

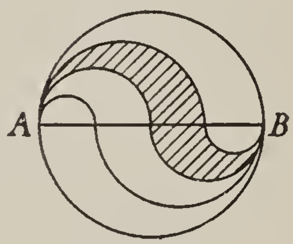


Fig. 190

7. Draw the design shown in Fig. 190, making AB equal to 8 centimeters.

8. Draw a *spiral* (Fig. 191).

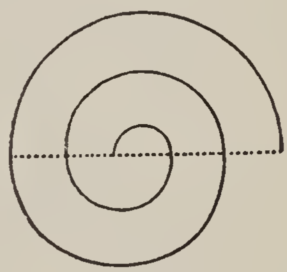


Fig. 191

9. Using ruler and protractor, draw a square whose side is 4 centimeters. Construct the designs shown in Figs. 192 and 193.

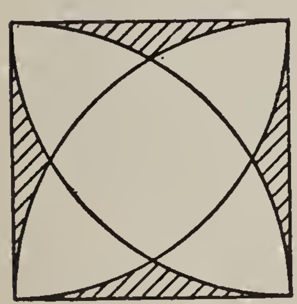


Fig. 192

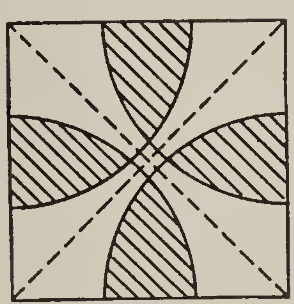


Fig. 193

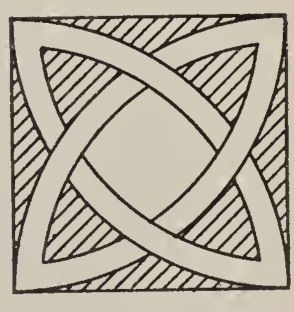


Fig. 194

10. Make a design like the one shown in Fig. 194.

11. Draw the design (Fig. 195) using your own ideas as to shading.

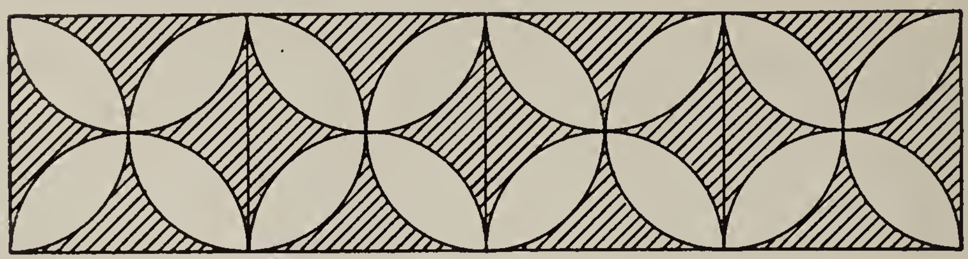


Fig. 195

12. Designs like those shown in Figs. 196 to 204 were made by high-school pupils. Make a copy of one of them. Try to make an original design as good as these, or better.

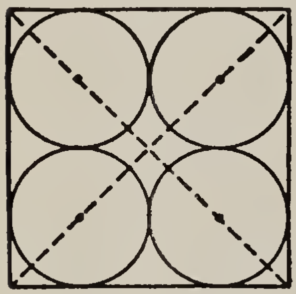


Fig. 196

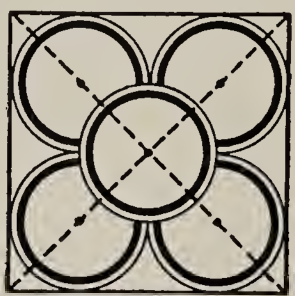


Fig. 197

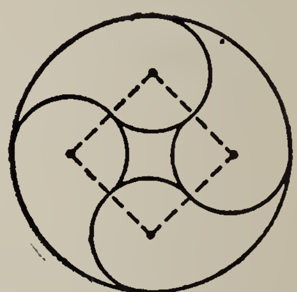


Fig. 198

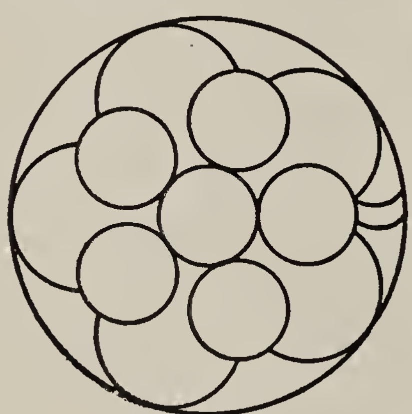


Fig. 199

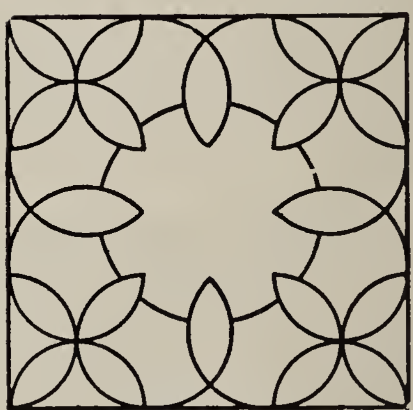


Fig. 200

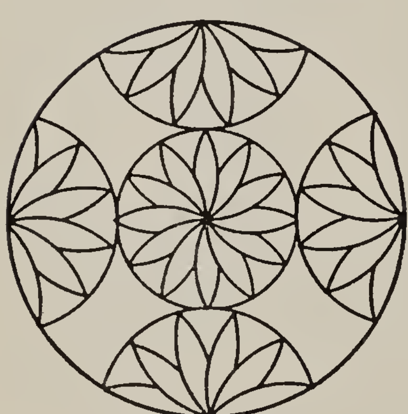


Fig. 201

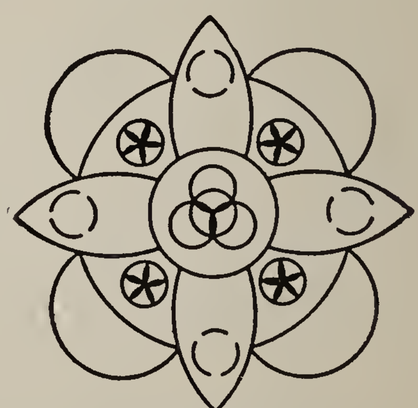


Fig. 202

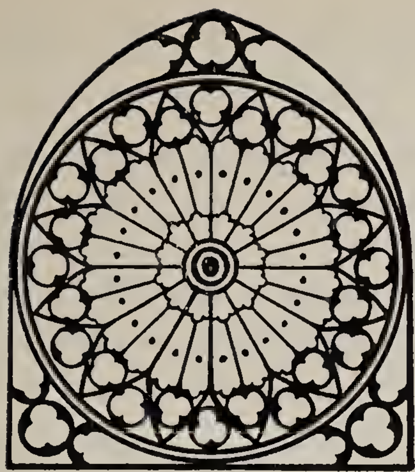


FIG. 203

FIG. 204

13. Draw an equilateral triangle.

Construction: Draw a segment AB (Fig. 205).

With A as center and radius AB draw an arc, as shown at C.

With B as center and radius AB draw a second arc intersecting the first at C.

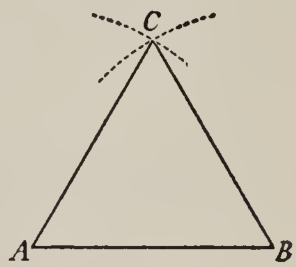


FIG. 205

Draw AC and BC.

Triangle ABC is the required equilateral triangle.

This construction is important because it is used in many designs, *e.g.*, in Fig. 206.

FIG. 206. GOTHIC WINDOW

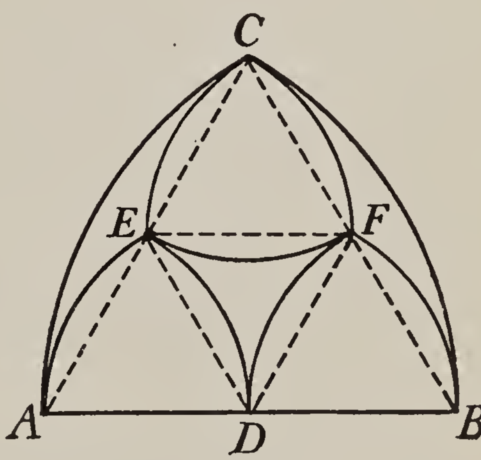


FIG. 207. DESIGN FOR GOTHIC WINDOW

14. Fig. 206 is a picture of a Gothic window. The design is based on the *Gothic arch* (Fig. 207), as ACB and AED. The construction is the same as that of Fig. 205. The arcs AC and BC are

drawn with B and A as centers and a radius equal to AB. Make a drawing of the arch.

15. The range of the guns of a fort is 10 miles. By a scale drawing show the ground covered by them.

16. Two forts are 18 miles apart. The range of their guns is 12 miles. Make a scale drawing showing the ground covered by the guns of the two forts.

17. A cow is tied by a rope 24 feet long to a stake placed at the mid-point of the longer side of a shed 12 feet by 10 feet in dimensions. Make a drawing showing the ground on which the cow may graze.

Uses of the Circle in Latitude, Longitude, and Time

103. Equal circles. Draw two circles using the same radius, one on ordinary notebook paper and the other on thin tracing paper. Place the tracing-paper circle over the other circle, and show that the two circles can be made to *fit exactly* (coincide). If two circles can be made to coincide they are *equal* circles.

The experiment above illustrates the fact that *two circles are equal if they have equal radii.*

Two equal circles may be considered as the same circle in two different positions. Hence, *radii of equal circles are equal.*

104. Central angles. Draw two equal ⓢ B and B_1 (Figs. 208 and 209), one on notebook paper and the other on thin tracing paper. Draw angle B having the vertex at the center B, and draw angle B_1 equal to angle B.

Place the tracing-paper circle (Fig. 208) on Fig. 209 so that point B falls on point B_1. Then $\odot B$ coincides with $\odot B_1$. Why?

Make $\angle B$ coincide with $\angle B_1$.

Point A must then coincide with A_1, and C with C_1; for radii of equal circles are equal.

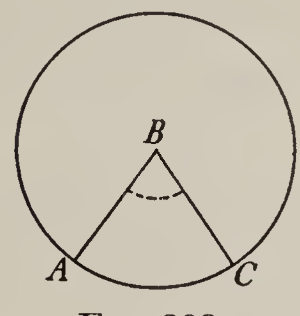


Fig. 208

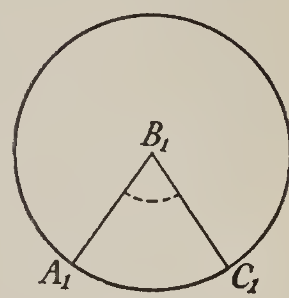


Fig. 209

Therefore arcs AC and A_1C_1 coincide, and are *equal.*

An angle having the vertex at the center of a circle is a **central angle.**

The discussion above shows that in *equal circles equal central angles cut off (intercept) equal arcs.* The same is true for two *equal* central angles in the *same* circle.

Exercises

1. Review carefully the preceding proof until you can repeat it correctly.

2. Show, as in §104, that *equal arcs, in equal circles, have equal central angles.*

105. Theorem. *A statement to be proved,* such as the statement proved in §104, is called a **theorem.**

106. How to measure angles by means of circle arcs. Draw a circle, and at the center B (Fig. 210) draw an angle, as ABC.

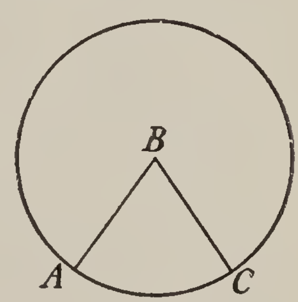


Fig. 210

Measure angle ABC with the protractor and note the number of degrees it

contains. Each degree in angle ABC cuts off (intercepts) a small arc on the circle, called **arc-degree.**

State the number of arc-degrees for a central *right* angle; a *straight* angle· a *perigon;* a 60-degree angle; a 45-degree angle.

What part of a circle is an arc-degree?

Arc-degrees are used to *measure* arcs. Since there are as many angle-degrees in a central angle as there are arc-degrees in the arc, we say that a central angle and the arc it cuts off have the *same measure*, or that *the central angle is measured by the intercepted arc.*

Exercises

1. If an arc contains 90 arc-degrees, what is the measure of the central angle of the arc?

2. State the measure of the central angles if the arcs contain 20°; 15°; 180°; 270°.

107. Quadrant. Semicircle. One-fourth of a circle is a **quadrant.** One-half of a circle is a **semicircle.**

108. Using the circle in locating places. Knowledge of the division of the circle into arc-degrees enables us to locate places exactly. This may be seen from the following. A man asked to be directed to a certain building in Chicago. He was told to walk three blocks west and then two blocks north. This illustrates a method by which places in a city are commonly located. A similar method is used in geography to locate places on the surface of the earth. However, in place of streets or roads, imaginary circular lines are laid out running from north to south and from east to

Fig. 211

west, and covering the surface of the earth. The north-south lines are called **meridians,** the east-west lines **parallels.** Figs. 211 and 212 represent the surface of the earth and show some of these circular lines. Thus circle *ABCDEF* (Fig. 212) passes around the earth midway between the poles P and P_1. This line is the **equator.** It divides the surface of the earth into the *Northern* and *Southern Hemispheres.* It is divided into 360 arc degrees (Fig. 213).

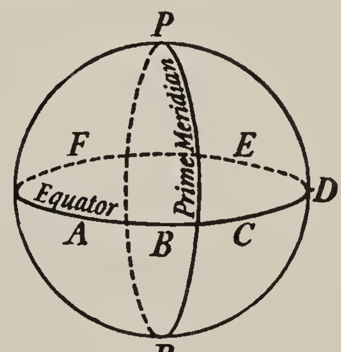


Fig. 212

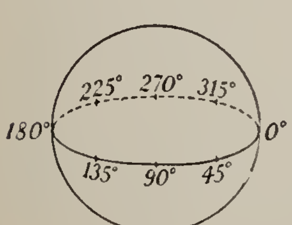


Fig. 213

The equal circles passing through the poles are the **meridians** (Fig. 214). The meridian which passes through the Royal Observatory at Greenwich, England, is the *prime meridian.* This meridian divides the surface of the earth into the *Eastern* and *Western Hemispheres.*

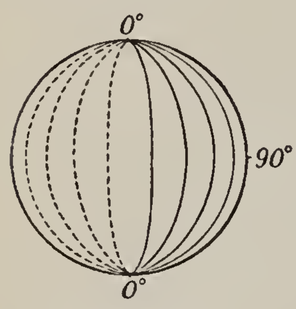


FIG. 214

109. Longitude and latitude. The number of degrees *east or west* of the *prime meridian* is called **longitude.** All places *east* of the prime meridian have *east longitude,* all places *west* have *west longitude.*

The number of degrees measured along a *meridian,* northward or southward from the equator, is called **latitude.**

Thus, the longitudes of places may vary from 0° to 180°, the latitudes from 0° to 90°.

For example, the location of the custom house at Portland, Maine, is: Lat. 43° 39′ 28″ N., Long. 70° 15′ 18″ W. To represent the location of Portland by a drawing, make a sketch of the Western Hemisphere (Fig. 215). From the 0° mark on the equator, pass along the equator to the left to point A, which is 70° 15′ 18″ W., and from A toward the north to point P, which is 43° 39′ 28″ north of the equator. Then the location of P is: Lat. 43° 39′ 28″ N., Long. 70° 15′ 18″ W. Find the latitude and longitude of your city and make a drawing like Fig. 215 to show the location.

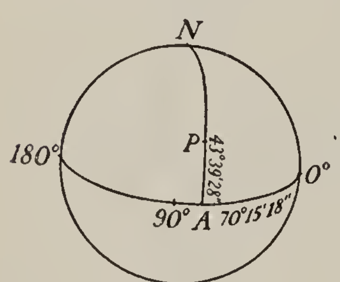


FIG. 215

Exercises

1. Practice making good drawings like Figs. 213 to 215.

Using the method given on page 160, make sketches locating the places in the table below:

Place	*Lat.*	*Long.*
2. Portsmouth, N. H., Navy-yard Flagstaff.	43° 4′ 56″ N.	70° 44′ 22″ W.
3. Boston, Mass., State House	42° 21′ 28″ N.	71° 3′ 50″ W.
4. Annapolis, Md., Naval Academy Observatory	38° 58′ 53″ N.	76° 29′ 8″ W.
5. Vera Cruz, Mex., Lighthouse	19° 12′ 30″ N.	96° 7′ 57″ W.
6. Staten Island, Argentina, Cape St. Bartholomew, Middle Point	54° 53′ 45″ S.	64° 45′ 45″ W.
7. Christiania, Norway, Observatory	59° 54′ 44″ N.	10° 43′ 23″ E.
8. Hamburg, Germany, Marine Observatory .	53° 32′ 52″ N.	10° 58′ 21″ E.
9. Melbourne, Australia, Observatory	37° 49′ 53″ S.	144° 58′ 35″ E.

10. On an outline map locate the places in the above table from the directions given in Exercises 2 to 9.

11. From a map find the approximate positions of Washington, D. C.; San Francisco; Chicago; Liverpool; Paris; Berlin; Rome.

12. A ship in Long. 10° W. sails west 16 degrees. What is the new longitude?

13. A ship starts in Long. 47° E. and sails 83° west. What is the new longitude?

Find the differences between the longitudes of important buildings in the following cities:

14. London, St. Paul's Cathedral, 0° 5′ 38″ W., and New York, City Hall, 74° 0′ 24″ W.

Solution:

New York	74° 0′ 24″	= 73° 59′ 84″
London	0° 5′ 38″	= 0° 5′ 38″
Difference		= 73° 54′ 46″

15. San Francisco 122° 25′ 42″ W. and Boston 71° 3′ 50″ W.

16. Newark, N. J., 74° 10′ 0″ W. and Manila 120° 58′ 6″ E.

17. Pittsburgh 80° 2′ 0″ W. and Cape Town 18° 28′ 45″ E.

110. Time. The earth may be regarded as a large sphere which turns around an imaginary line passing through the poles. It makes a complete revolution, or turns 360°, in 24 hours.

It is *noon* at a certain place when the meridian which passes through it is *directly under the sun.*

In one hour this place has passed over an arc equal to $\frac{1}{24}$ of 360°, or 15°. Why?

In one minute it has passed over $\frac{1}{60}$ of 15°, or 15′.

In one second it has passed over an arc equal to 15″.

Using the fact that 360° of longitude correspond to a complete turn, prove the following corresponding values of *time* and *longitude:*

Longitude......	360°	15°	15′	15″	1°	1′	1″
Time..........	24 hr.	1 hr.	1 min.	1 sec.	4 min.	4 sec.	$\frac{1}{15}$ sec.

EXERCISES

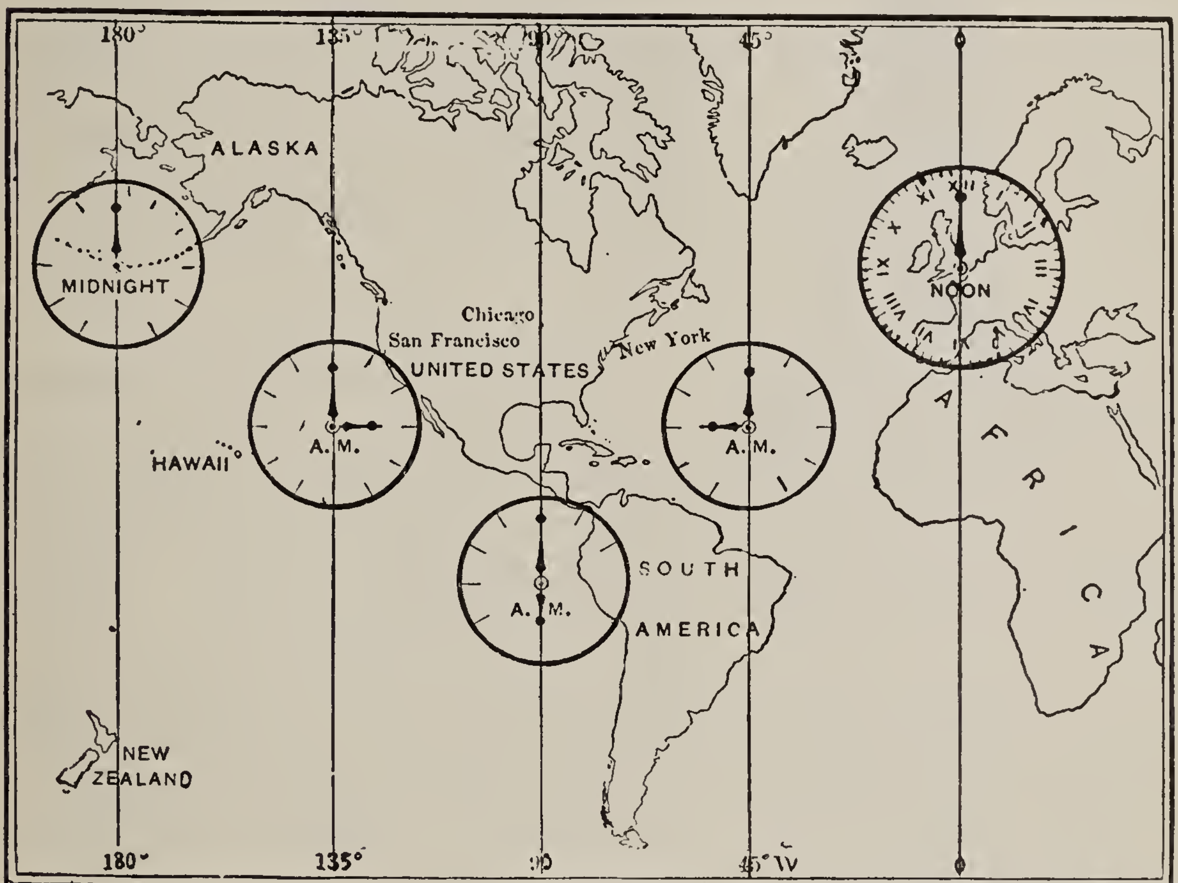
1. When it is 10 A.M. in Chicago, determine the corresponding local time of a place 22° east of Chicago; 18° west of Chicago; 25° east of Chicago.

Suggestion: To 1° of longitude correspond 4 minutes of time. Hence to 22° correspond (22×4) minutes.

2. Find the number of degrees of longitude corresponding to 8 seconds of time; 10 minutes; 30 minutes; 42 minutes.

3. Find the time corresponding to a change in longitude of 20°; 10′; 2° 15′.

111. Local time. Since the earth turns from west toward east, the objects in the sky, which may be



regarded as fixed in position, seem to pass over us from east toward west. Therefore if it is noon where we are, it is *earlier*, or *before* noon, at all places *west* of us, but it is *later*, or *after* noon, at all places *east* of us.

Thus for all places having the same longitude, *i.e.*, lying on the *same* meridian, the time is the *same;* but it is different for places on different meridians. Time reckoned this way is called **local time.**

Years ago people used to check the accuracy of their watches by comparison with a sun dial. Some of the larger cities distributed time by dropping a time-ball (Fig. 216) situated on a high tower to indicate exact noon. Today it is not necessary for a person to compare his watch directly. Time is computed by astronomers and distributed all over the country by telegraphic transmission.

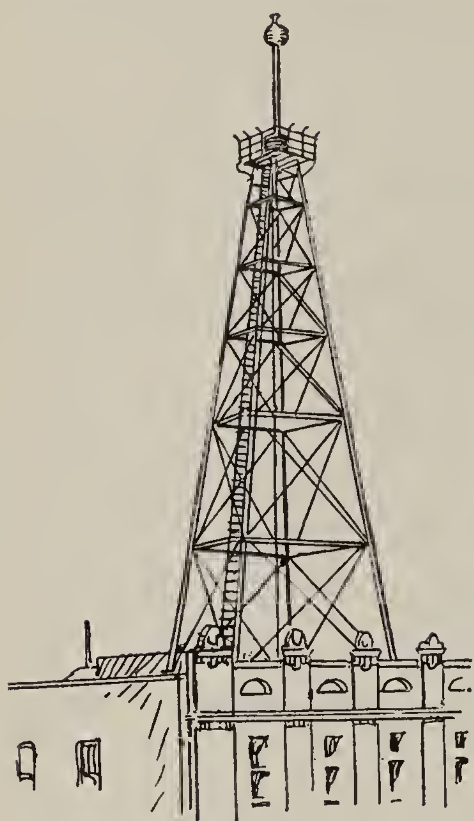


Fig. 216. Note the Time-Ball on Top of the Tower

Exercises

1. Important events happening in France during the war were read in the St. Louis dailies at an earlier hour than was indicated as the time of sending the cable message. Explain how this happened.

2. The longitude of certain buildings in Chicago and New York are 87° 36′ 42″ W. and 74° 0′ 3″ W., respectively. When local time is noon at New York, what is local time at Chicago?

Solution:

Difference in longitude = 13° 36′ 39″.

Using the table in §110, show that this is equal to

$$(13\times4)\text{ min.}+(36\times4)\text{ sec.}+\tfrac{39}{15}\text{ sec.}$$
$$=52\text{ min.}+2\text{ min.}+24\text{ sec.}+2.6\text{ sec.}$$
$$=54\text{ min.}+26.6\text{ sec.}$$

∴ The local time at Chicago is 54 min., 26.6 sec. before noon.

The table below gives the longitude of some important buildings of several large cities:

Places	*Washington*	*San Francisco*	*Boston*	*Denver*
Longitude . .	77° 3′ 0″ W.	122° 25′ 42″ W.	71° 3′ 50″ W.	104° 58′ W.

3. On a steamer crossing the Atlantic the clocks are moved ahead 32 minutes a day. What is the change in longitude each day?

4. A man going abroad finds that after several days spent on the ocean his watch is 3 hours and 42 minutes slow. By how much has the ship's longitude changed?

5. When it is 8 A.M., local time, in Washington, find the time in Boston.

6. When it is noon in Denver, find the time in San Francisco.

7. When it is 3 P.M. in Denver, find the time in Washington.

8. The time in Denver is 2 hours 3 minutes 56 seconds earlier than in New York. Find the difference in longitude.

9. Two ships at sea are 68° 12′ 25″ of longitude apart. What is the difference in time?

112. Standard time. When it was customary to use only local time, each locality had to determine its own time. This was satisfactory as long as travel was slow and infrequent. The telegraph and railroads brought about changes, however, which made it necessary to regulate the time in a more satisfactory way than formerly. A person traveling east or west had to set his watch continually to have the local time of the cities at which he was stopping. Because railroads were inconvenienced in the same way, the principal railroads of the United States adopted a uniform system of keeping time. They divided the country into four

time belts (Fig. 217). These belts are approximately 15° wide. The dividing lines are irregular and depend largely on the location of railroad terminals. With each belt the time changes one hour. The **Eastern Belt**

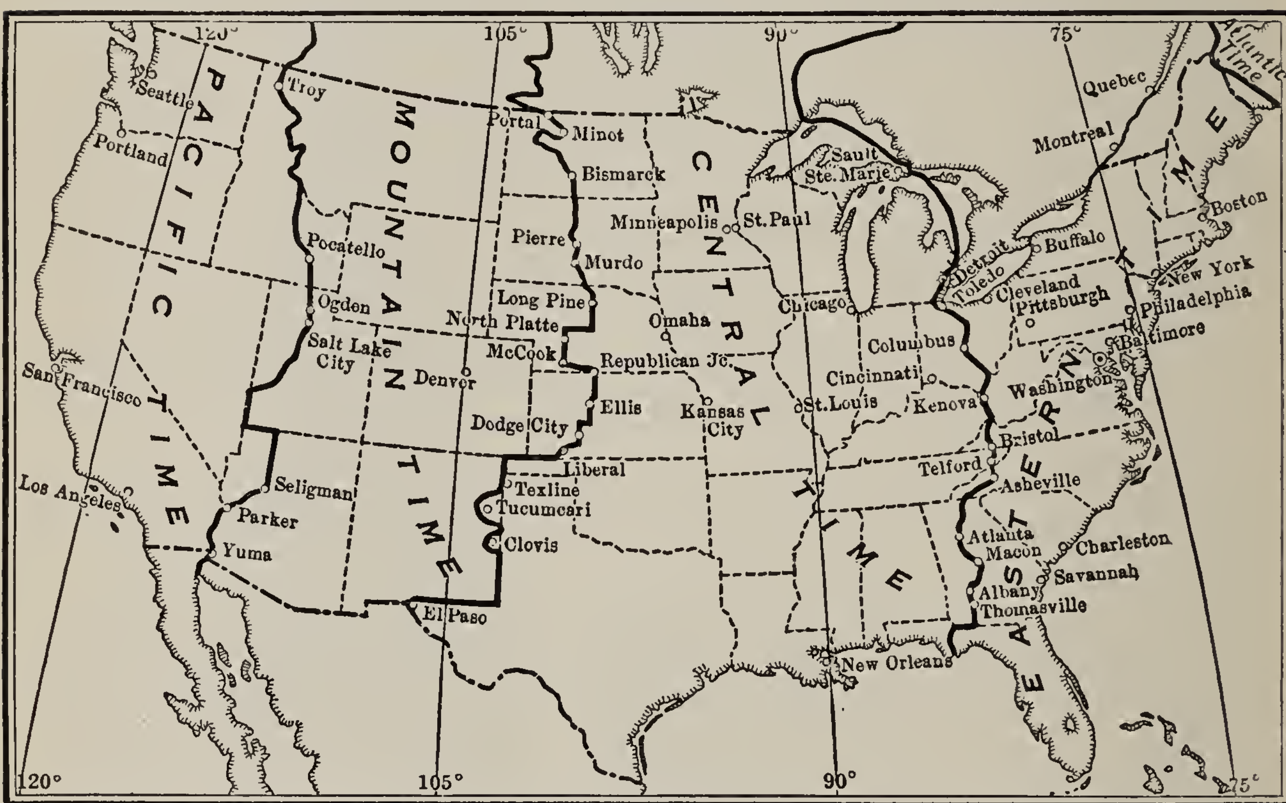


Fig. 217

extends on both sides of the 75th meridian and all places within it have the same time as the local time of the 75th meridian. The **Central Belt** has the local time of the 90th meridian. The **Mountain Belt** and **Pacific Belt** take the time of the 105th and 120th meridian respectively.

Exercises

1. How much faster are the clocks in any time belt than the clocks in the next time belt to the west?

2. From Fig. 217, find some cities which have approximately the same local time.

3. When it is 6 o'clock A.M. standard time in Chicago, find by using Fig. 217 the time in New York, St. Louis, Denver, San Francisco.

4. Try to find out what other countries have adopted the use of time belts, and what longitude determines the time in such countries.

113. Daylight saving. Standard time was introduced because it removed the inconvenience caused by the continuous change of local time to persons traveling east or west. Changes in time may be desirable for other reasons. During the World War, when coal became scarce because of the large amounts used in factories supplying the government with war materials, an arbitrary change in time was made to help preserve the coal supply. The clocks were moved one hour ahead. With this change a person who ordinarily retires at 9 P.M. really retires at 8 P.M., and gains an additional hour of daylight without losing an hour in the morning. The result is a saving of light and coal.

Some people liked this plan very much. It made the electric-light bills a little less for the family, and considerably less for factories and large stores. It lengthened the day during the summer months, giving the working people an extra hour of recreation in daylight and enabling those who had gardens to spend time on them after school or working hours.

Some people objected to the plan, especially the farmers. They were always early risers, but with daylight saving they had to rise even earlier in order to complete the necessary amount of work and to get their products to the trains at the right time. The extra hour of daylight proved to be a hardship to them.

People who traveled were greatly inconvenienced because the railroads did not adopt daylight saving, which made railroad time differ from daylight-saving time used in various cities.

Another objection to the daylight saving plan comes from mothers and doctors, who say that small children cannot get the proper amount of sleep, because under the new plan the children must be put to bed before dark.

However, in spite of these objections some cities continued the daylight-saving plan even after the war.

Exercises

1. Some tourists left the hotel at 8:30 A.M. to take a train. When they reached the station 10 minutes later, the clock in the waiting room gave the time as 7:40 A.M. How do you explain this if both clocks gave the correct time?

2. We left Chicago at 10:00 A.M., daylight-saving time. Having traveled at a moderate speed we stopped in a city in Indiana 49 miles from Chicago to have lunch. According to the clock in the restaurant it was only 10:45 A.M. How fast did we travel?

Uses of the Circle in Measuring Public Utilities

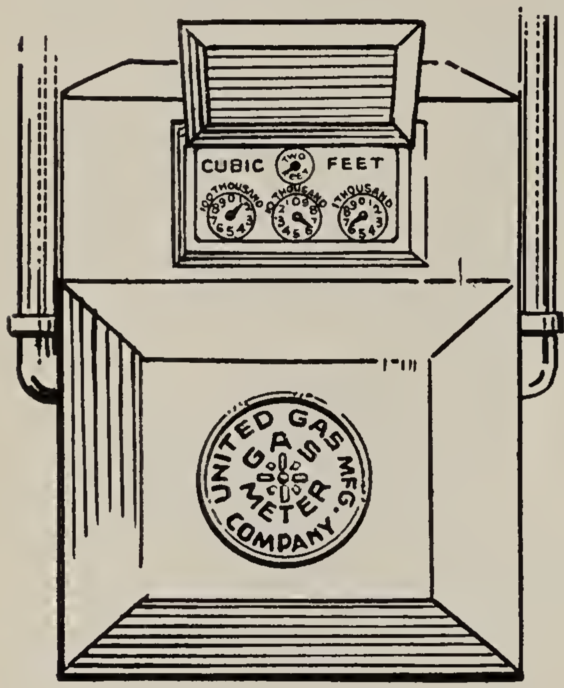
114. The need of accurate measurement. During the World War when it was difficult to secure help, gas companies of the larger cities had to employ many incompetent workers for their meter service. This resulted in thousands of complaints from citizens who claimed that their gas bills were too high for the amount of gas used. Errors in gas bills were due, mainly, to carelessness or ignorance in reading meters and in

recording the readings, and to inaccurate calculation of the bills by the bookkeepers. At all times users of public utilities should be able to check the correctness

of their gas and electric-light bills. They should know how to read a meter and be able to calculate the correct amount to be paid to the company.

115. How gas is measured. In many cities gas used for cooking and illuminating is made from coal.

It is kept in large tanks from which it is forced into pipes leading to the houses of the consumers.



The cost of gas is usually fixed at a certain sum per 1000 cubic feet. The number of cubic feet used is recorded by the *gas meter*. The amount is shown by means of circles (dials) divided into 10 equal arcs (Fig. 218). Each division on the right-hand circle denotes 100 cu. ft.; on the center circle 1000 cu. ft.; on the left-hand circle 10,000 cubic feet.

As gas is being used the hands turn in the direction 0, 1, 2, etc.

The middle dial is numbered in a direction reverse to that of the other two.

When reading a meter, always note the figure just passed by the hand. The meter is read from the left-hand dial to the right. In Fig. 218 the first hand has passed over 6, the second over 5, and the third over 7. Accordingly, the reading is 65,700, which means that 65,700 cu. ft. of gas have been used.

FIG. 218

Exercises

Read the following meters:

1.

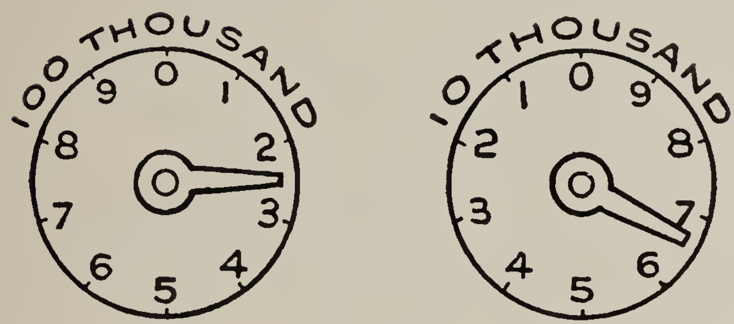


Fig. 219

2.

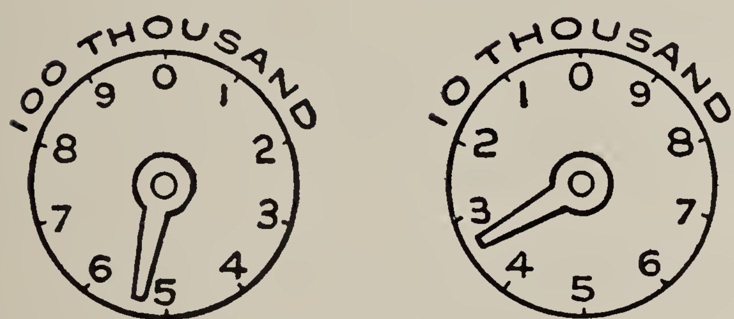


Fig. 220

3.

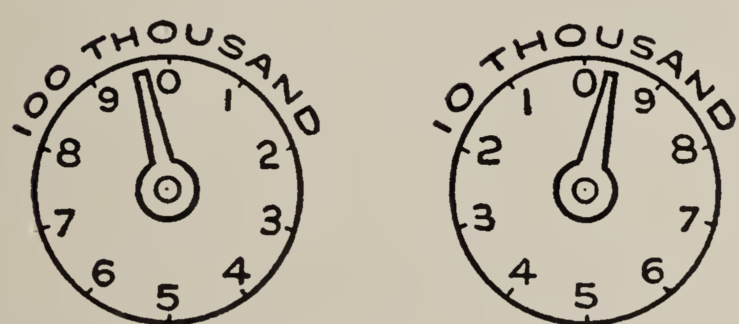


Fig. 221

4.

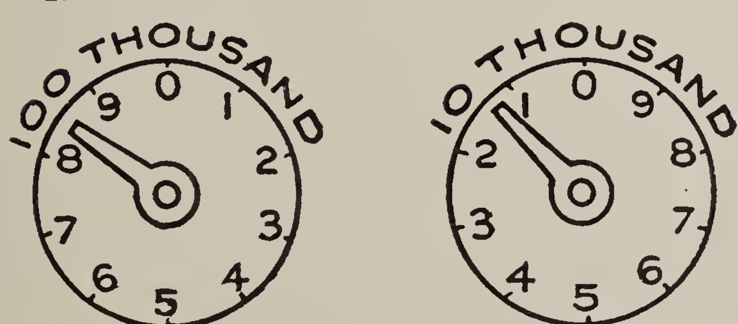


Fig. 222

116. How to check the correctness of a gas bill. The cost of gas in a city is 85 cents per 1000 cubic feet. According to his gas bill, the reading of Mr. Young's meter on Jan. 7 was 46,300. A month later it was 52,800. Hence, the amount of gas used during the month was 52,800 − 46,300, or 6500, cubic feet.

GAS BILL OF FEB. 18, 1924 **GROSS AFTER FEB. 28**

PLEASE BRING THIS BILL WHEN PAYING AT OUR OFFICE

7130 Lawrence Ave. 147
J. W. Young 72
1

TO **THE PEOPLES GAS LIGHT & COKE CO.,** DR.

METER READINGS AND GAS USED IN CUBIC FEET SHOWN BELOW	For CUSTOMER'S RECORD Paid by ______ Check No. ______ Bank ______ Date ______
Jan. 7 46300	GROSS 5.52
Feb. 7 52800	LESS 10c PER M CU. FT. .55
6500	NET 4.97
	PREVIOUS GAS BILL ______ ______
RATES: $.85 per M net	" " " ______ ______
	MERCHANDISE LEASE NO. ______ ______
	" " " ______ ______
If not paid on or before Feb. 28, 1924, the gross amount is due and payable. 1	TOTAL AMOUNT DUE

Thus, to find the amount of gas used within a certain time, *one notes the reading of the meter at the end of that time and subtracts from that amount the reading taken at the beginning.* The result gives the number of cubic feet of gas used during the intervening time.

To *find the cost* proceed as follows:

On Feb. 7, the reading is 52,800 cu. ft.
On Jan. 7, the reading is 46,300 cu. ft.

Hence the number of cubic feet of gas consumed = 6500 cu. ft.

At \$.85 per M., the gas bill for the month amounts to

$$\$\left(\frac{6500}{1000}\times .85\right) = \$(6.500\times .85) = \$5.52$$

Computation:

$$\begin{array}{r} 6.500 \\ .85 \\ \hline 32500 \\ 52000 \\ \hline 5.52500 \end{array}$$

The decimal point is fixed by adding the number of places in the decimals of the factors. It may also be determined by estimating: Since $6\times .8$ is 4.8, or approximately 5, the decimal point should be placed after the 5.

The product is 5.52500, or 5.52, the last three figures being dropped.

Exercises

Find the exact products in the exercises below:

1. 3.26×8.41.	**4.** 9.49×3.84.	**7.** 84.61×32.03.
2. 8.23×14.2.	**5.** $.746\times .0028$.	**8.** 18.362×3.1415.
3. 3.12×5.68.	**6.** 5.081×36.05.	**9.** $32.41\times .613$.

10. In a certain city, the gas company supplies gas at 90 cents per 1000 cubic feet. Find the cost of gas used by four different consumers if the readings of their meters were as shown in the table below. Arrange the work as shown above.

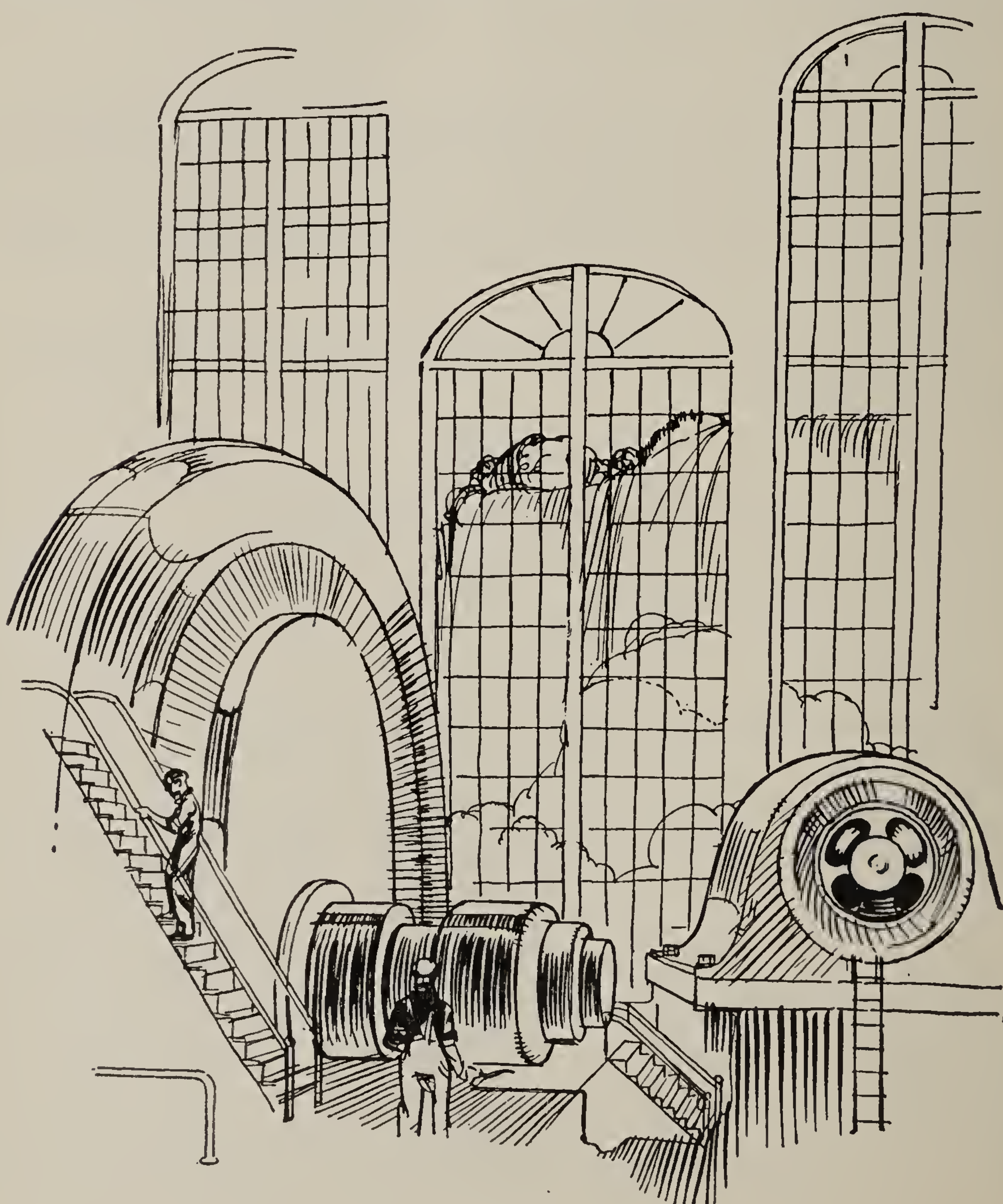
	Last Reading	*Previous Reading*	*No. of Cu. Ft. Gas Used*	*Cost*
I.	56,700	23,500		
II.	78,600	54,800		
III.	92,400	86,300		
IV.	95,300	88,200		

11. Make a sketch of a gas meter whose reading shows 43,500 cubic feet.

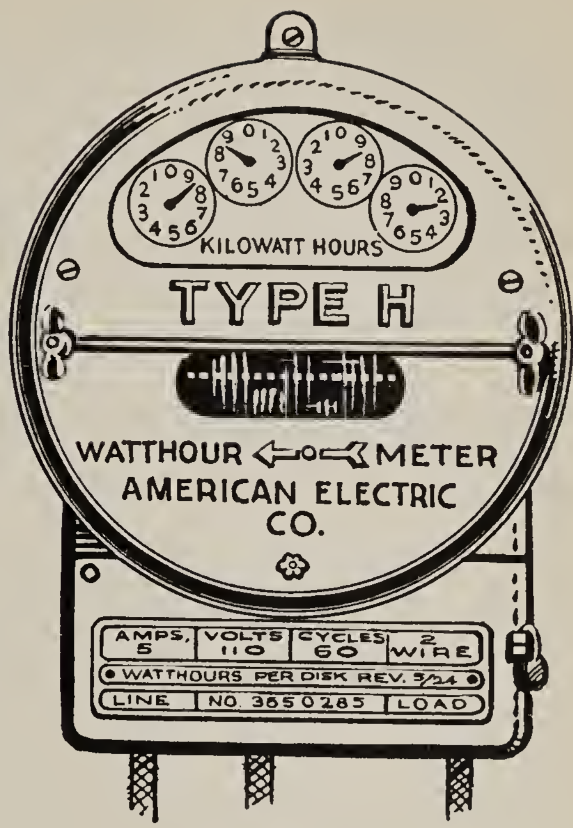
117. Measuring electricity. Electricity is measured by means of electric meters in terms of *watt hours* (W.H.) and in *kilowatt hours* (K.W.H.).

1000 watts = 1 kilowatt

The dials of an electric meter (Fig. 223) are like those of the gas meters. They show, respectively,



thousands, hundreds, tens, and units. The divisions of the dials from right to left denote 1 kilowatt hour, 10 kilowatt hours, 100 kilowatt hours, and 1000 kilowatt hours. When reading the meter one must first note the direction in which each pointer is turning and then the figure just passed over by the pointer. Thus the meter (Fig. 223) reads 3457 kilowatt hours. Why?



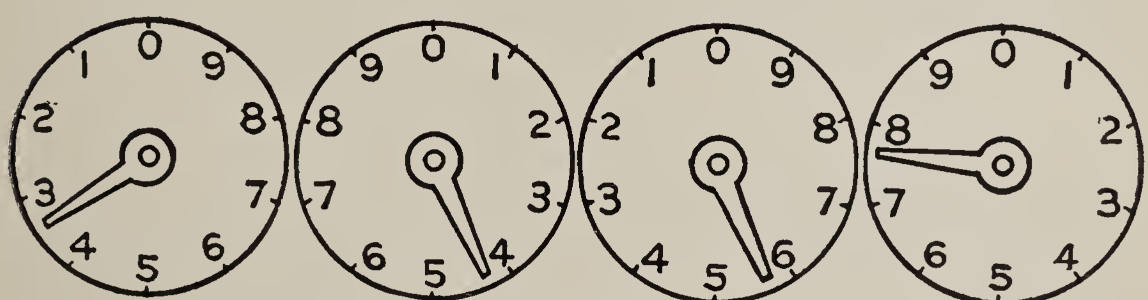


Fig. 223

Exercises

1. State the reading of each of the following electric meters (Figs. 224 and 225):

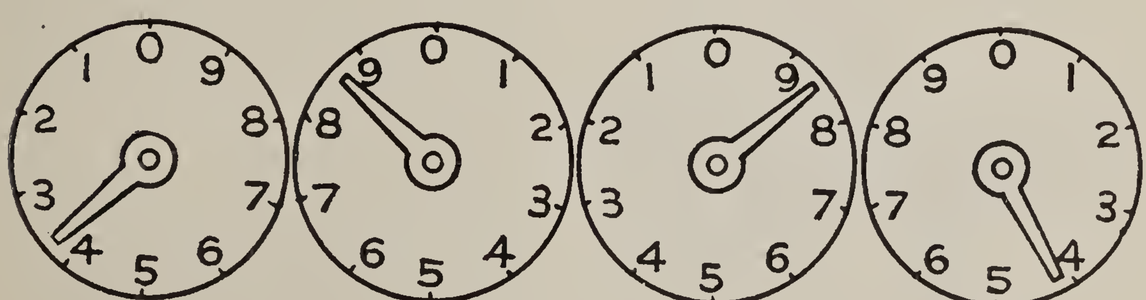


Fig. 224

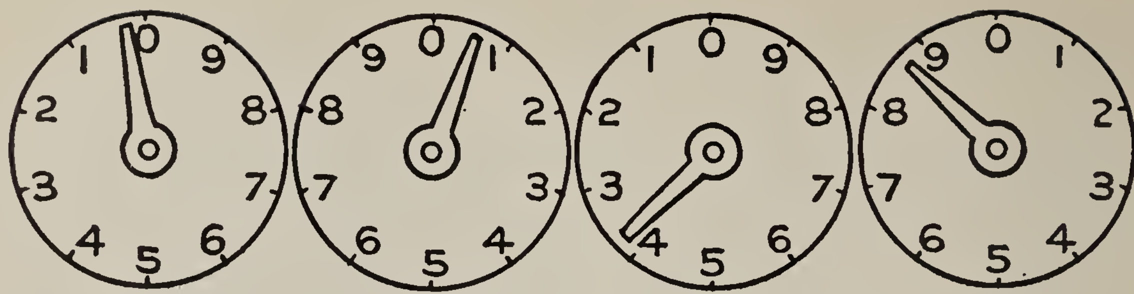


FIG. 225

2. Make a sketch of the dials of a meter showing a reading of 6875 kilowatt hours.

USES OF THE CIRCLE IN GRAPHICAL REPRESENTATION

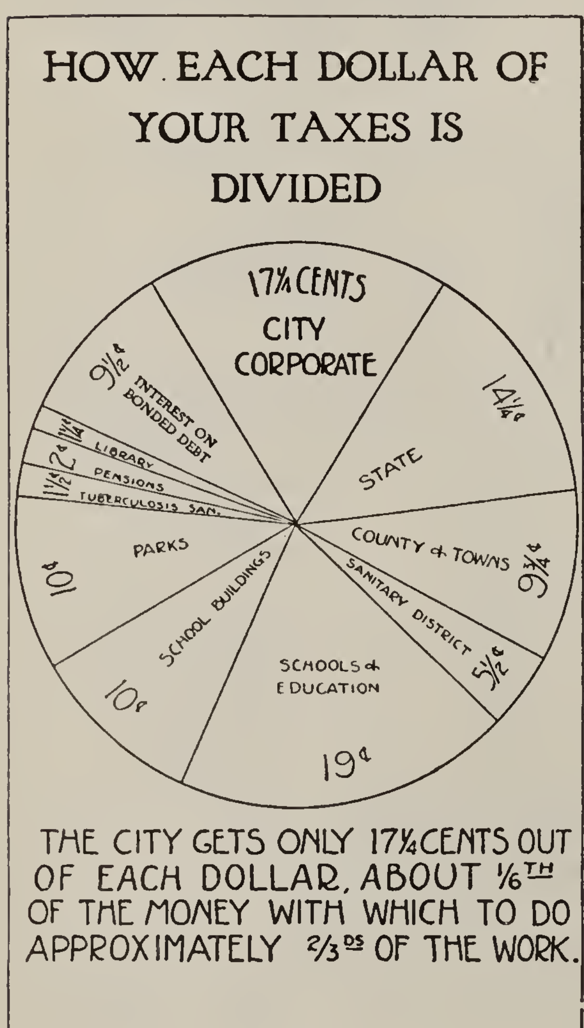


FIG. 226

118. How to read circular graphs. The picture (Fig. 226) represents graphically the distribution of the taxpayers' money in one of our large cities (Chicago) so that one may see at a glance how the money has been divided. The various amounts are represented as parts of the interior of a circle. This is an excellent device for comparing any item with each of the others and with the sum of all. Such a graphical device is called a **circular graph.**

The graph represents the numerical facts given in the following table:

	Cents
For interest on bonded debt	9.50
Library	1.25
Pensions	2.00
Sanitarium	1.50
Parks	10.00
School buildings	10.00
Education	19.00
Sanitary district	5.50
County	9.75
State	14.25
City	17.25
Total Tax	100.00

Exercises

The growth of the foreign-born population has raised serious problems in this country. Figs. 227 to 230 illustrate graphically the relative proportions of our foreign-born as found in the years 1870, 1890, 1910, and 1922. The changes that have taken place suggest interesting questions. Study these graphs, and discuss the changes in the foreign-born population for the various countries. For what country is the increase largest? For what country is the decrease largest? For what country is the change least? Can you find explanations for these changes?

Make a study of Figs. 227 to 230, and tell what these graphs show.

FIG. 227. OUR FOREIGN-BORN POPULATION IN 1870.

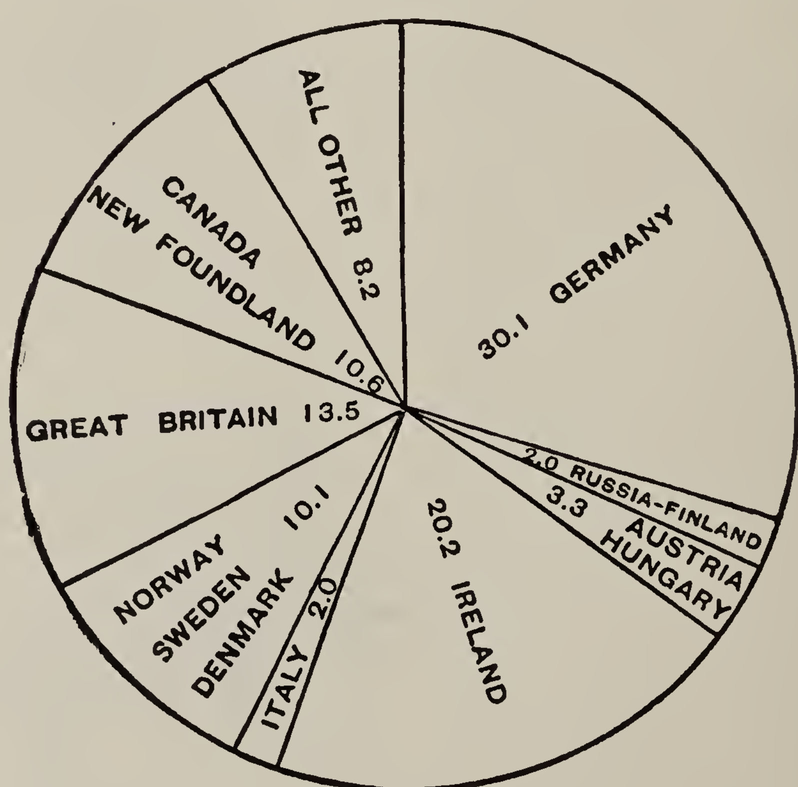


FIG. 228. OUR FOREIGN-BORN POPULATION IN 1890.

Fig. 229. Our Foreign-Born Population in 1910.

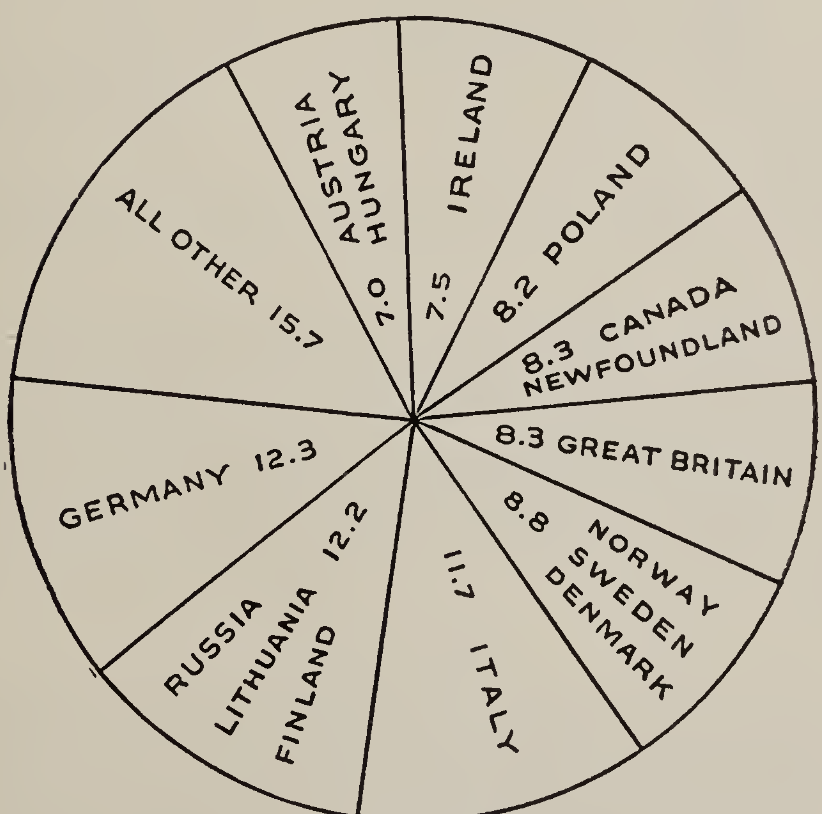


Fig. 230. Our Foreign-Born Population in 1922.

119. Making a circular graph. To make a circular graph from a given table of numerical facts, proceed as follows:

1. *Divide each of the items in the table by the sum of all.* For example, to show what portion of the money was used for education (Fig. 226), divide 19 by 100.

2. *Divide the circle in the same ratio.* Let n be the number of degrees in the required arc representing education. Then

$$\frac{n}{360}=\frac{19}{100}=.19.$$

3. *Solve the equation.*

Multiplying both numbers by 360, we have

$$\frac{\cancel{360}n}{\cancel{360}}=(.19)\ (360)$$

Hence, $n=68.4$.

4. With a protractor draw a central angle equal to 68.4°.

5. Similarly, find the number of degrees in the arcs corresponding to the other items in the table. Add the results as a check of accuracy, and draw the angles in a circle as in Fig. 226. Mark each item in the corresponding angle.

Exercises

1. The income of a certain family, in which everybody has to help pay expenses by earning some money, is as follows: The father earns \$1400, the mother \$27 by doing special work, and the children \$180. Boarders contribute \$144, and \$45 comes from

other sources. The mother, who has to manage the household, must spend the money wisely and carefully.

She plans her expenditures as follows: She allows for food and fuel $936, for rent $324, and for clothing $216. The remainder is set aside for sickness, insurance, education, furniture, savings, and incidental expenses. Using the method given on page 180, construct a circular graph representing the incomes earned by the members of the family, and one representing the expenditures.

2. In a school of eight grades, the number of pupils in each grade is as follows:

Grades	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*Total*
No. of pupils	32	27	26	24	22	18	8	6	163

Represent the facts in this table by means of a circular graph.

3. In 1920 the population, in millions, in the United States, was as stated below:

Native-born whites	81.1
Foreign-born whites	13.7
Negroes	10.5
Others	.4
Total	105.7

Represent these facts by means of a circular graph.

4. The sales of a store for 1924 amounted to $45,000. The cost of the goods sold amounted to $30,000, and the expenses were $10,000. The remainder was profit. Illustrate these facts by means of a circle graph.

5. The tax-rate table below shows how the money obtained from taxes is divided in various sections of a large city. Following the suggestions given on page 180, make a circular graph for each.

TAX–RATE TABLE

	State	*County*	*City*	*School*	*Sanitary*	*Forest Preserve*	*Park*	*Lincoln Pk. Bond*	*Town*	*Total*
South	.40	.49	2.08	1.59	.23	.05	.30			$5.14
West	.40	.49	2.08	1.59	.23	.05	.52			5.36
North	.40	.49	2.08	1.59	.23	.05	.44	.06	.07	5.41

6. The table below states some reasons why children leave the public schools. Represent the facts by a circle graph.

Causes	*Number of Pupils Withdrawn*
Leaving city	1,153
Going to work	284
Poor health	195
Staying at home	121
Going to private school	84
Various other causes	41

7. A street car company gave wide publicity to the following statement:

Where your 8-cent car fare goes	*Cents*
1. Wages	4.12
2. Power, materials	1.49
3. City's share	0.46
4. Interest	0.95
5. Taxes and damages	0.54
6. Company's share	0.44
	8.00

Represent these facts by means of a circular graph.

8. In a family one of the daughters keeps a careful account of all expenditures. At the end of the year she finds that she has used $1078.23 of her parents' money. This amount was distributed as follows:

Food	$583.75	Car fare	$12.45
Clothing	342.82	Doctor	43.00
Furniture	83.71	Dentist	12.50

Make a circular graph representing these facts.

9. Find illustrations of circular graphs in books on science, geography, business, cooking, and other subjects. Look for uses of circular graphs in newspapers and magazines. Then write a paper on circular graphs.

120. What all pupils should know and be able to do. Having studied Chapter VI every pupil should be able:

1. To copy simple designs using only ruler and compass, as shown in §102; to draw an equilateral triangle;
2. To use correctly the terms: local time, standard time, latitude, longitude, time belt, and circle;
3. To change longitude to local time;
4. To read gas and electric-light meters;
5. To make circular graphs;

The following theorems should be known:

1. Equal circles have equal radii.
2. Equal central angles have (intercept) equal arcs.
3. Circles having equal radii are equal.
4. Equal arcs have (are intercepted by) equal central angles.
5. A central angle is measured by the intercepted arc.

121. Typical problems and exercises. Give answers and solutions for the following questions and problems:

1. Explain how you read a gas meter. Illustrate your statement with a drawing, showing a reading of 54,800 cubic feet.

2. How are places on the surface of the earth located by means of latitude and longitude? On a drawing locate the position of a place whose latitude is 54° 42′ S., and whose longitude is 76° 30′ W.

3. What is the relation of local time to longitude? When it is 1 P.M. in Denver, what is the corresponding local time of a place 32° east of Denver?

4. How is the difference in local time for two cities found from the longitudes?

5. A store sells $83,000 worth of goods in the year, the cost of which was $52,000. The expenses amounted to $12,000. Make a circular graph showing cost, expenses, profits, and total sales.

6. Write a paper on one of the following topics:
   a. Designs made by means of circles.
   b. Uses of the circle.
   c. Time.
   d. Daylight saving.

CHAPTER VII

GEOMETRIC CONSTRUCTIONS. MEASUREMENT OF THE CIRCLE

The Use of the Circle in Making Constructions

122. Geometric constructions. The constructions in this chapter are to be made by using only compass and straightedge. Such constructions are called *geometric constructions.* The graduated ruler and the protractor may be used as checking instruments after the construction is made.

123. Construction of a triangle whose sides are given. One of the most useful applications of the circle is the construction of geometrical figures. A simple construction is the drawing of a triangle whose sides are to be of given lengths (Exercise 13, §102). This construction is made as follows:

Draw a triangle, as ABC (Fig. 231).

Draw $A_1D > AB$, and with the compass lay off A_1B_1 on A_1D making $A_1B_1 = AB$.

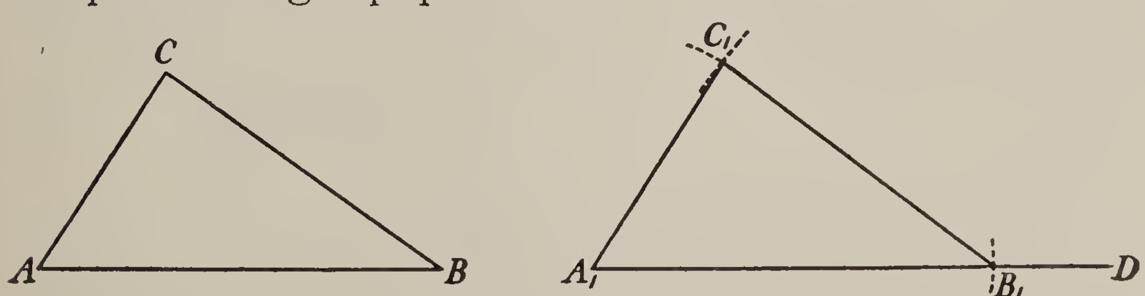


Fig. 231

With radius AC and A_1 as center, draw an arc at C_1.

With radius BC and B_1 as center, draw a second arc intersecting the first at C_1.

Draw A_1C_1 and B_1C_1.

Cut $\triangle A_1B_1C_1$ from the paper and place it on ABC.

If your construction is well done it will be possible to make the two triangles coincide.

This exercise illustrates the following principle: *When two triangles have the corresponding sides equal they are congruent.*

The principle just established means that all triangles having their corresponding sides equal are really the same triangle in different positions, and cannot be different in size or in shape. This is the reason why triangles are used in the construction of objects that are to stay rigid, such as trusses, brackets, etc. (Fig. 232).

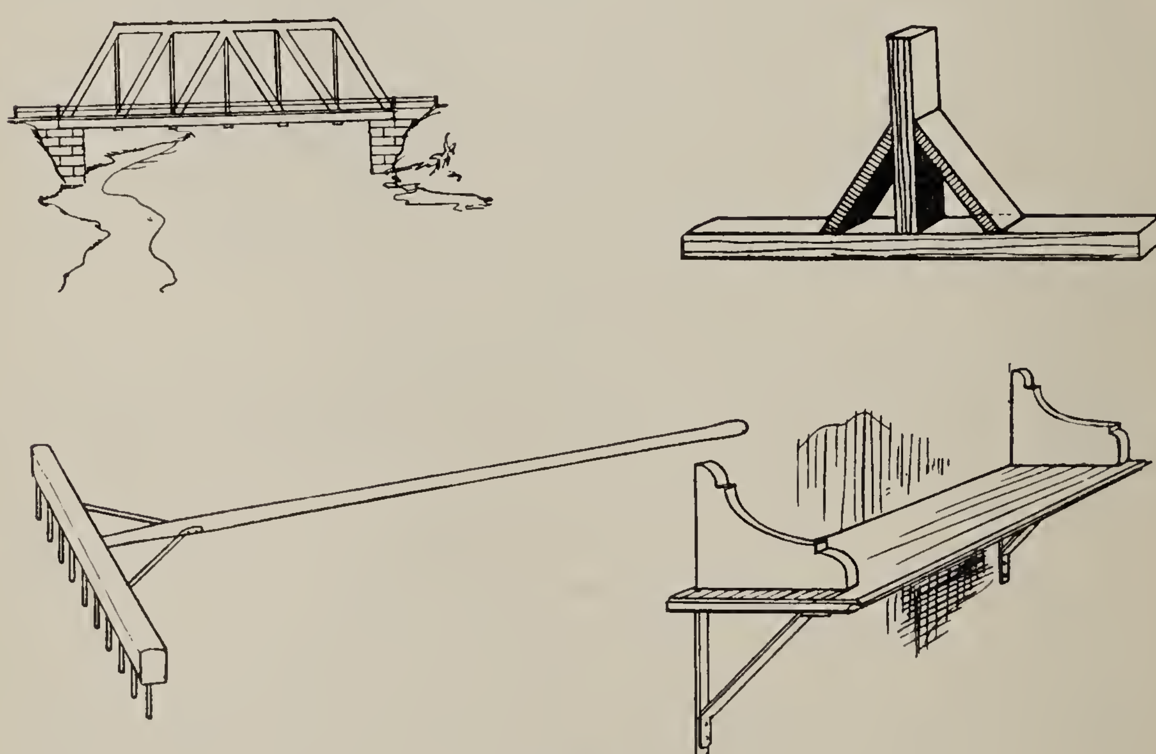


Fig. 232

Trusses are networks of triangles. They are used in the construction of roofs and bridges. Since a triangle is a rigid figure, a network of two or more triangles is also rigid.

124. Bisecting an angle. Draw $\angle ABC$ (Fig. 233) to represent any given angle.

With B as center and a convenient radius, draw arcs cutting AB and CB in points D and E.

With D and E as centers and a radius longer than one-half the distance DE, draw arcs intersecting at F.

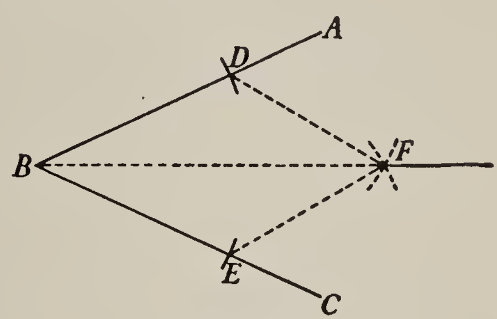


Fig. 233

Draw BF.

Line BF divides angle ABC into two equal parts. To test the accuracy of your construction, measure $\angle s\ ABF$ and CBF with the protractor.

It is easier to *prove* the equality of these angles than to show it by measuring. For $\triangle s\ FDB$ and FEB have the corresponding sides equal by construction. They are therefore congruent (§ 123). It follows that the corresponding angles FBD and FBE are equal.

When a line drawn through the vertex of an angle divides the angle into two equal parts, it **bisects** the angle. The dividing line is the **bisector** of the angle.

125. Constructing an angle of 60°. We have seen that the angles of an equilateral triangle are equal to each other, and each equal to 60°. Hence, to construct

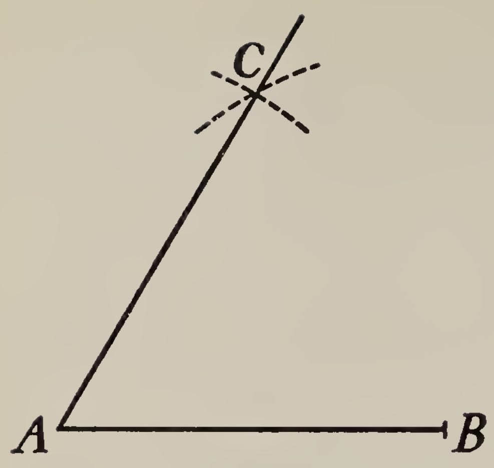


Fig. 234

an angle of 60° proceed as in the construction of an equilateral triangle (Exercise 13, §102).

Draw two arcs (Fig. 234) meeting at C, using AB as radius and A and B as centers.

Then draw AC.

$\angle BAC$ is the required 60°-angle.

Exercises

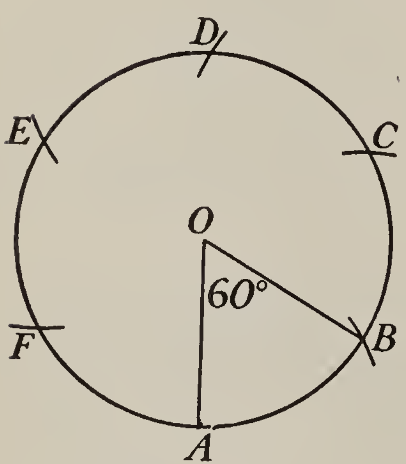


Fig. 235

1. Construct an angle of 30°; of 15°.

Suggestion: Use §124.

2. Divide a circle into 6 equal parts.

Suggestion: At the center draw an angle equal to 60° (Fig. 235).

Then $AB = \frac{1}{6}$ of the circle. Why?

With radius equal to OA and B as center, draw an arc at C.

Similarly, draw arcs at D, E, and F.

3. Draw the following designs (Figs. 236 to 241) using compass and straightedge only:

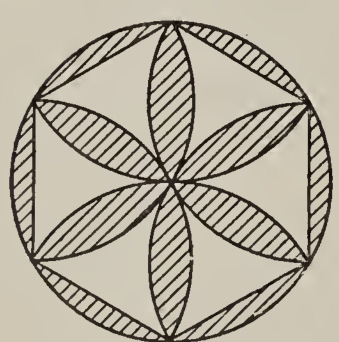


Fig. 236

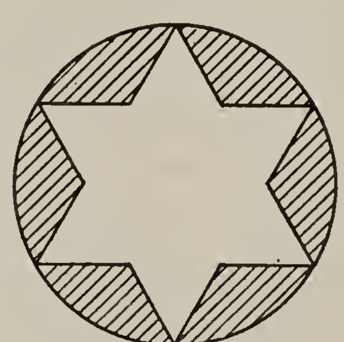


Fig. 237

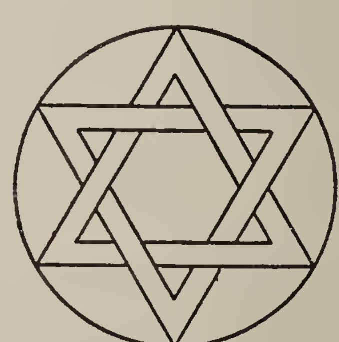


Fig. 238

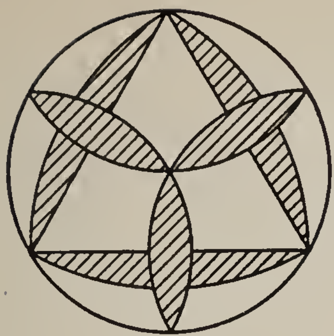


FIG. 239

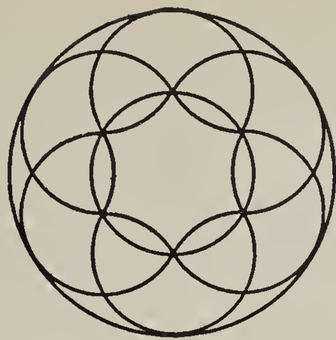


FIG. 240

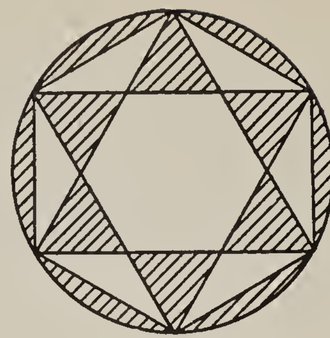


FIG. 241

126. Construct a regular hexagon. Divide a circle (Fig. 242) into six equal arcs (Exercise 2, §125).

Join the points of division, forming the hexagon *ABCDEF*.

This is the required hexagon.

The correctness of the construction may be proved as follows:

Proof: Draw radii OA, OB, OC, ..., OF.

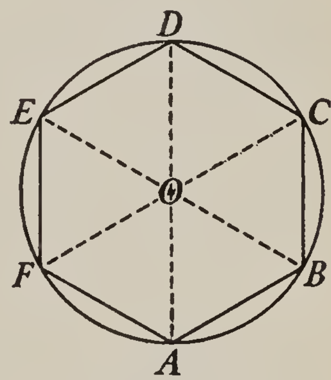


FIG. 242

Show that the six triangles formed are congruent and equilateral.

It follows that the sides AB, BC, CD, etc., are equal to each other.

Prove that $\angle FAB = 120°$.

Similarly, $\measuredangle ABC$, BCD, CDE, DEF, and EFA are each 120°.

Therefore $\angle ABC = \angle BCD = \angle CDE =$ etc. Why?

It has been shown that (1) the hexagon *ABCDEF* is *equilateral* and (2) *equiangular*. A polygon which is equilateral and equiangular is called a **regular polygon.**

Hence *ABCDEF* is a regular hexagon.

A polygon all of whose vertices lie on a circle is an **inscribed polygon.**

Exercises

1. Draw an equilateral inscribed triangle.

Suggestion: Divide a circle into 6 equal parts and draw 3 line segments joining alternate points of division.

2. Construct an angle of 120 degrees.

Suggestion: Draw two 60-degree angles on opposite sides of the same segment.

3. Draw a regular 12-sided polygon (dodecagon).

Suggestion: Draw a regular inscribed hexagon. Bisect the arcs by drawing the bisectors of the angles at the center.

127. Bisect a line segment. Let AB (Fig. 243) be the segment to be bisected.

Using A as center and a convenient radius, draw arcs above and below AB.

With the same radius and center at B, draw arcs cutting the first two arcs at C and D.

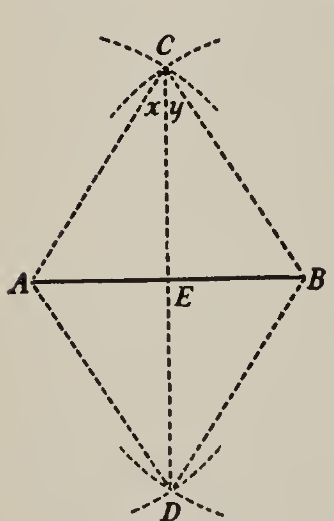


Fig. 243

Draw CD intersecting AB at E.

Then AB is bisected at E,

$$i.e., AE = EB.$$

We may *prove* that

$$AE = EB \text{ as follows:}$$

$\triangle CAD \cong \triangle CBD$, because the sides of one triangle are respectively equal to the sides of the other.

It follows that $x = y$. Why?

Then $\triangle ACE \cong \triangle BCE$, because two sides and the included angle of one are equal to the corresponding parts of the other.

$$\therefore AE = EB$$

Furthermore, $CD \perp AB$, because the two adjacent angles at E are equal.

Since CD is *perpendicular* to AB and *bisects* AB, it is the *perpendicular bisector* of AB.

Exercises

Construct the following figures, using only straightedge and compass:

1. Draw an equilateral triangle (Fig. 244). Bisect the sides and complete the figure as shown.

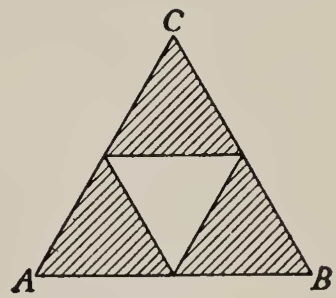


Fig. 244

2. Draw a triangle and bisect each side. Join each of the points of bisection to the opposite vertex.

3. Draw Figs. 245 to 247.

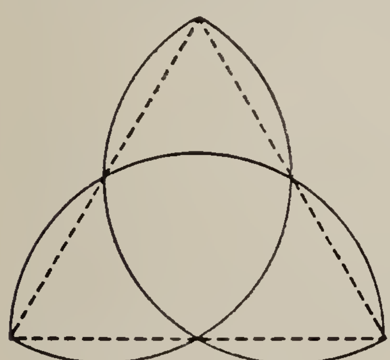


Fig. 245

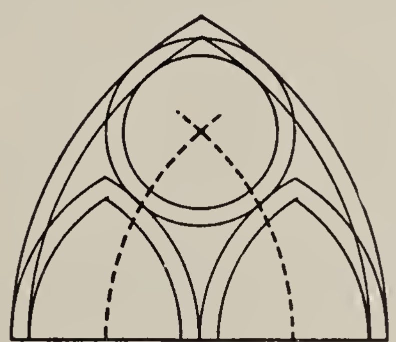


Fig. 246

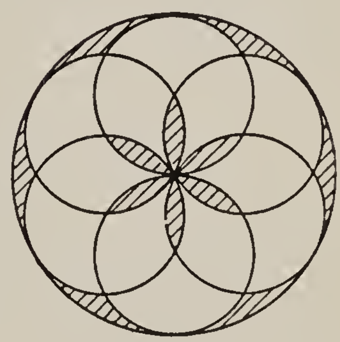


Fig. 247

4. Draw a triangle ABC. Construct the perpendicular bisectors of the sides.

If your construction is made carefully, the three bisectors will meet in a point. Using the point as a center, draw a circle passing through the vertices of $\triangle ABC$.

5. Make the construction of Exercise 4 for a triangle having an obtuse angle; for a triangle having a right angle.

128. Construction of perpendicular lines. Let it be required to construct a line perpendicular to the segment AB (Fig. 248) at the point C.

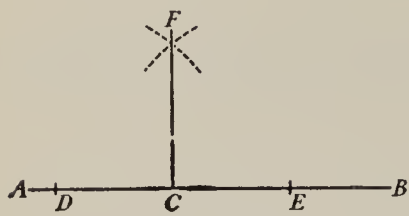


FIG. 248

Using C as center and a convenient radius, draw arcs at D and E.

Using D and E as centers and a radius greater than CE, draw arcs intersecting at F.

Draw FC.

Then FC is the required perpendicular to AB.

Test the accuracy of your construction with the protractor.

We may *prove* that CF is perpendicular to DE by first showing $\triangle DCF \cong \triangle ECF$.

It then follows that the two adjacent angles at C are equal.

$$\therefore CF \perp DE$$

EXERCISES

1. Construct angles of 90°, 45°, and 135°, using only compass and straightedge.

2. Construct a square, using only compasses and straightedge.

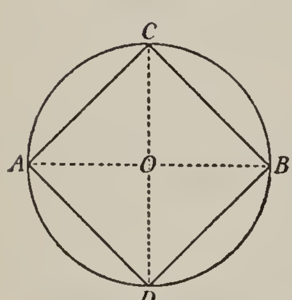


FIG. 249

3. Construct a square inscribed in a circle.

Suggestion: Draw a diameter, as AB (Fig. 249).

Draw diameter $CD \perp AB$.

Draw AD, DB, BC, and CA.

$ADBC$ is the required square.

4. Construct Figs. 250 to 254, using only compass and straightedge. Make the diameter of the large circle 3 inches.

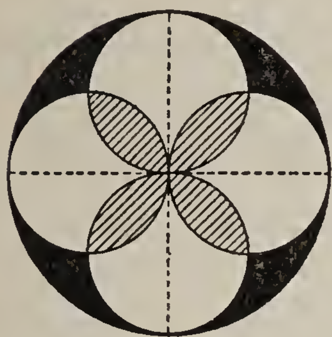


Fig. 250

Fig. 251

Fig. 252

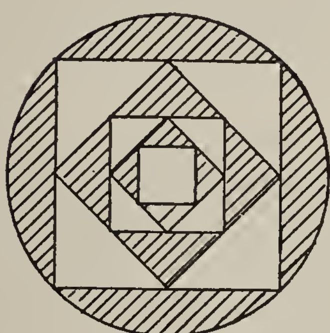


Fig. 253

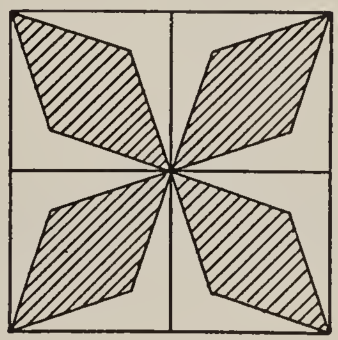


Fig. 254

5. Draw a regular inscribed octagon (8-sided polygon).

Suggestion: Draw two diameters perpendicular to each other. Then draw the bisectors of the angles at the center intersecting the circle. Join these points of intersection.

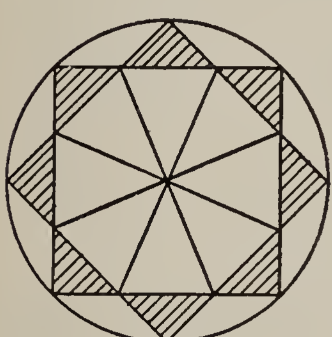


Fig. 255

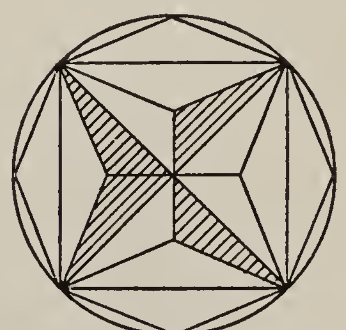


Fig. 256

6. Make the designs shown in Figs. 255 and 256.

129. Draw a perpendicular to a line from a point not on the line. Draw AB (Fig. 257). Select a point C not on AB.

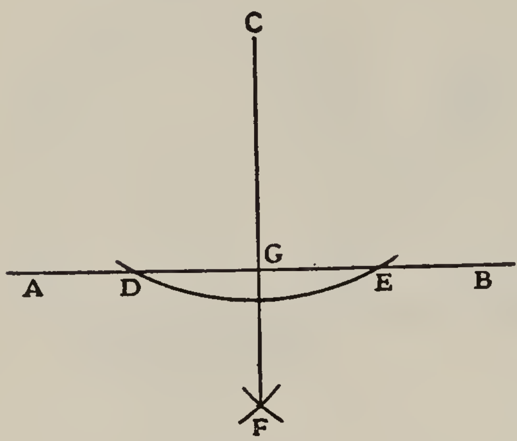


Fig. 257

With C as center and a convenient radius, draw an arc intersecting AB at points D and E.

With D and E as centers, draw arcs intersecting at F.

Draw CF.

CF is the required line.

Test the accuracy of your construction with the protractor.

130. What is meant by distance from a point to a line. The length of the perpendicular from a point to a line, as CG (Fig. 257), is called the **distance from the point to the line.**

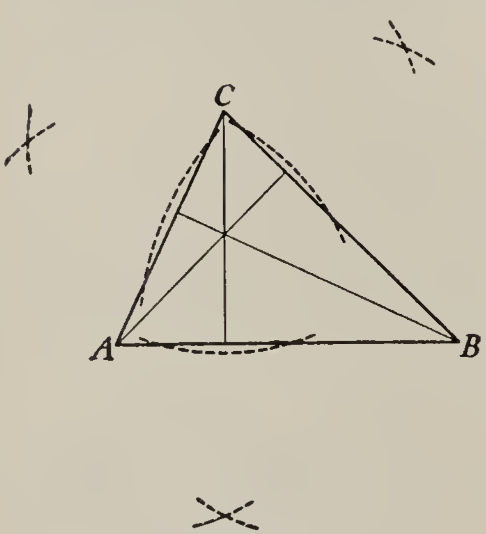


Fig. 258

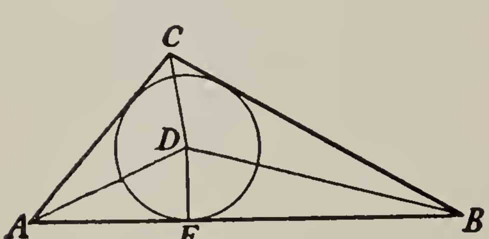


Fig. 259

Exercises

1. Draw a triangle whose angles are all acute (Fig. 258). From each of the points A, B, and C construct a line perpendicular to the opposite side.

If the drawing is accurate, the three perpendiculars meet in one point.

Make the construction for a triangle having an obtuse angle.

2. Draw a triangle (Fig. 259). Bisect the angles.

If accurately drawn, the three bisectors intersect in the same point.

From the point of intersection D of the bisectors of the angles draw DE perpendicular to AB.

Using DE as radius and D as center, draw a circle.

If the construction is carefully made, the circle just touches the sides of the triangle.

3. Draw a triangle ABC. Extend two of the sides, AB and AC, as shown in Fig. 260. Bisect the exterior angles at B and C and the interior angle at A.

Denote the point of intersection of these three lines by O. Using O as center and the perpendicular from O to BC as radius, draw a circle.

If the drawing is accurate, the circle touches BC and the extensions of AB and AC.

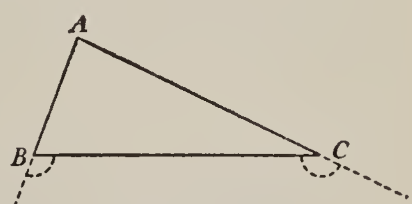


Fig. 260

131. Altitude of a triangle. The perpendicular from the vertex of a triangle to the opposite side (§130) is an **altitude** of the triangle.

132. Concurrent lines. Lines passing through the same point are **concurrent** lines; *e.g.*, the altitudes, the bisectors of the angles, and the perpendicular bisectors of the sides of a triangle are *concurrent* lines.

133. Constructing an angle equal to a given angle. Draw $\angle ABC$ (Fig. 261).

Draw a segment DE.

With B as center and any radius, draw arcs cutting BA and BC at F and G respectively.

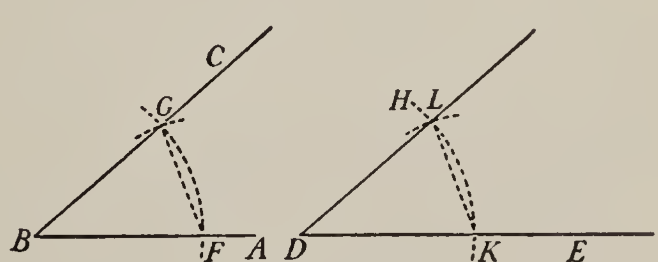


Fig. 261

With D as center and the *same* radius, draw arc HK.

With K as center and radius equal to FG, draw an arc cutting HK at L.

Draw DL.

$\angle KDL$ is the required angle.

Measure both angles with the protractor to test the accuracy of the drawing.

To *prove* that $\angle B = \angle D$, draw GF and LK.

Then prove $\triangle FGB \cong \triangle KLD$.

Exercises

1. Through B (Fig. 262) draw a line parallel to AC, using compass and straightedge only. See §70 for construction with the protractor.

Suggestion: At B draw an angle equal to $\angle A$ (§130).

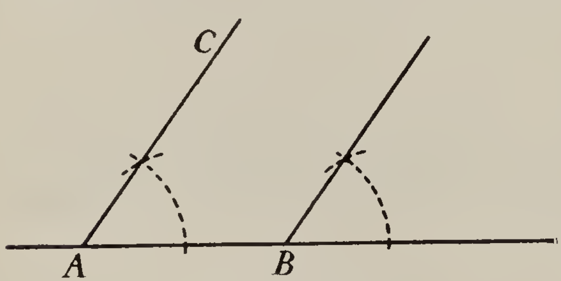


FIG. 262

2. Draw a triangle and bisect one side. Through the mid-point, draw a line parallel to another side of the triangle. If the drawing is accurate, the third side will be bisected by the parallel. Use this as a test of accuracy.

The Circumference of a Circle

134. Measurement of the circle. With a tape line, measure carefully the distance around a cylindrical jar or tin can, and record the result in the second column of a table as shown below.

Measure the distance around other circular objects, such as plates or phonograph records, and place the results in the table.

Cut a circular piece of cardboard, and measure the distance around the circle. Enter the result in the table.

The curved surface of the cylindrical object (Fig. 263) is bounded by circles. On one of the circles mark a point P. Place P on point A of a straight line AB, and roll the object, without sliding, so that the circle always touches the line, until point P touches line AB a second time, at C.

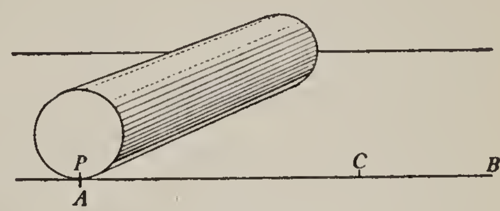


Fig. 263

The circle may then be measured by measuring AC. Record the result in the table.

Objects	*Distance Around*	*Diameter*	*Distance ÷ Diameter*
Glass jar.			
Baking-powder can. . . .			
Circular plate.			
Pipe.			
Glass.			
Half-dollar.			
Circle.			

Measure the diameters of the circles measured above, and record the lengths in the table.

For each object find the ratio of the length of the circle to the diameter. With careful measurements these ratios should all be the same.

Find the average of the ratios in the table.

135. Circumference. The length of a circle is called **circumference.** It has been proved by mathematics that *for all circles the ratio of the circumference to the diameter is the same.* The ratio, computed approximately to three figures, is 3.14, or $3\frac{1}{7}$. The exact value of the ratio is denoted by the Greek letter π (read

pi). It stands for the letter *p*, and is the first letter in the Greek word for perimeter. It came into use as a literal number in the middle of the eighteenth century. The fact that the ratio of the circumference to the diameter is the same for all circles was known to mathematicians long before that time, but they assigned to it different values. In one of the earliest books, written by Ahmes about 1700 B.C., π is given equal to $\frac{256}{81}$ or 3.1604. The Jews considered π equal to 3.

Denoting the circumference by c and the length of the diameter by d, we have the formula

$$\boldsymbol{c = \pi d}$$

Let r be the length of the radius of a circle. Show that

$$\boldsymbol{c = 2\pi r}$$

Translate these two formulas into words.

Exercises

In the following exercises use $\pi = 3.14$.

1. Mary wishes to make a circular lamp shade whose diameter should be 23 inches. How much fringe does she need if she allows 1 inch for waste in joining the ends?

Solution: This is really a problem of finding the circumference when the diameter is known.

$$d = 23$$

and $c = \pi d$

Taking π to three figures, we have

$$c = 3.14 \times 23,$$

or $c = 72.2$ approximately, the 2 at the right being in doubt.

Hence from 72 to 73 inches of fringe are required.

Computation:

Placing a dot over all doubtful figures, we have

$$\begin{array}{r} 3.1\dot{4} \\ 23 \\ \hline 9\ 4\dot{2} \\ 62\ \dot{8} \\ \hline 72.\dot{2}\dot{2} \end{array}$$

2. Helen's father is making a circular flower bed 12 feet in diameter. He has sent Helen to buy the plants which are to be placed along the border 6 inches apart. How many plants are needed?

3. Find the length of the equator, assuming the diameter of the earth to be 8,000 miles approximately.

4. The diameter of a circular grass plot is 56 feet. Find the distance around it.

5. What is the circumference of the largest circular table top that can be cut from a square whose side is 36 inches long?

6. How much lace will be needed to make an edge for a doily 24 inches in diameter, if we allow $\frac{3}{4}$ inch for the seam and 3 inches for fulling in?

7. What radius should be used to mark out a circular flower bed if you have 48 plants which are to be placed 6 inches apart to form the border?

Solution: $c = 48 \times 6 = 288$,

$c = \pi d = 3.14d$,

$\therefore 3.14d = 288$,

and $d = \dfrac{288}{3.14} = 92$ approximately.

Hence $r = \frac{1}{2} \times d = 46$,

$r = 46$.

Computation:

$$
\begin{array}{r}
91.7 \\
314\overline{)28800} \\
\underline{2826} \\
540 \\
\underline{314} \\
226 \\
\underline{2198}
\end{array}
$$

8. Find the diameter of a tree trunk at a height where the circumference is 74 inches.

Suggestion: Use the method of Exercise 7.

9. The circumference of a circle is 42 inches. Find the diameter and radius.

10. A circular pond is surrounded by a walk whose inner circumference is 98 feet. Find the diameter of the pond.

11. When ordering clothing from a mail-order house one must state the size. This is easily determined for some articles. The size

of a boy's coat is the number of inches of the chest measure. The size of a collar is the number of inches between the outer ends of the button holes. But the size of a shoe is not the length of the shoe, and clothes for a small boy or girl are usually ordered according to his or her age.

The size of a man's hat may be determined by measuring the distance around his head and dividing the result by $3\frac{1}{7}$.

If the distance around a man's head is $22\frac{3}{4}$ inches, what is the size of his hat?

12. Two athletes are running on a circular track. One is running 4 feet farther from the center than the other. How much

farther will he have to run? Work this out for various diameters, *e.g.*, 10, 15, 18.

13. The diameter of a wagon wheel is 42 inches. What distance does the wagon move when the wheel makes one complete revolution?

14. Explain why the speedometer of an automobile does not register correctly when oversized tires are put on the wheels.

15. Find the circumference and diameter of a wheel which makes 700 revolutions going one mile.

Suggestion: Show that $700c = 5280$.

16. Find, in feet, the distance traveled by a point on the rim of a fly-wheel which is 18 inches in diameter and makes 500 revolutions a minute.

17. The rear wheel of a wagon is 4 feet in diameter, and the fore wheel $3\frac{1}{2}$ feet. How many more revolutions than the rear wheel does the fore wheel make when the wagon travels one mile?

18. Denoting the circumference of a wheel by c, the distance it travels by l, and the number of revolutions it makes by n, make a formula which expresses l in terms of n and c.

19. By means of the formula $c=3.14d$, find, to 3 figures, the value of c if $d=1.26$; 4.08; 2.39; 18.2.

20. By means of the formula $c=\frac{22}{7}d$, find the value of c if $d=\frac{3}{5}$; $\frac{5}{6}$; $2\frac{1}{2}$; 4.6; $1\frac{5}{33}$.

21. By means of the formula $c=3.14\ d$, find the value of d if $c=6.20$; 8.42; 9.36.

Graphical Representation of the Formula $c=\pi d$

136. Relation between diameter and circumference. We have seen that by means of the relation $c=3.14d$, we are able to determine for any given value of d the corresponding value of c, *i.e.*, that the value of *c depends upon* the value of d. In the table below write the values of c corresponding to the values of d given in the first row:

d	1	2	3	4	5	6	7	8
c								

To represent graphically the facts stated in this table lay off, to a convenient scale, the values of d horizontally and the values of c vertically (Fig. 264).

From the graph find c when $d = 1\frac{1}{2}$; $2\frac{1}{2}$; $6\frac{1}{2}$.

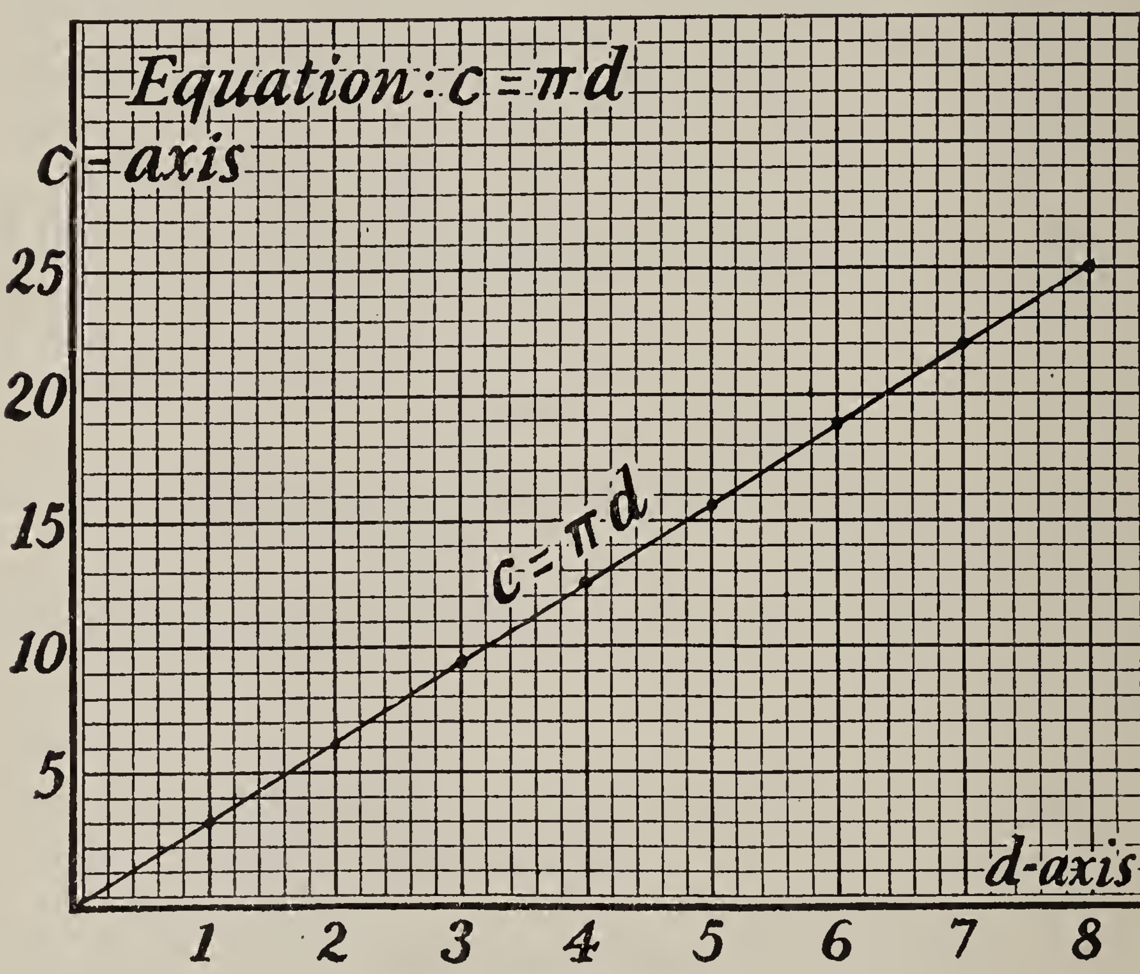


FIG. 264

137. Direct variation. To the table (§136), add a third line giving the ratio $\frac{c}{d}$ for each pair of values of c and d.

Notice that as we let d vary (change) from 1 to 8, c also varies. A change in d affects the values of c. For example, if d is doubled, c is also doubled. However, the ratio $\frac{c}{d}$ remains always the same. This law of

variation is known as *direct variation.* When we say *c varies directly as d,* we mean that c and d vary in such a way that *the ratio* $\frac{c}{d}$ *remains constant* (the same). The graph (Fig. 264) represents geometrically the relationship between c and d which is expressed algebraically by the formula $c = \pi d$.

This type of variation is very common. We meet it in a variety of problems, some of which will be studied in Chapter VIII.

138. What every pupil should know and be able to do.

All pupils should know how to make the following constructions with ruler and compass:

1. To draw angles of 60°, 30°, 15°, 120°, 90°, 45°, 135°.
2. To bisect an angle.
3. To bisect a segment.
4. To draw a perpendicular to a line at a point on the line; from a point not on the line.
5. To draw a line parallel to a given line.
6. To draw an angle equal to a given angle.
7. To divide a circle into 4 equal arcs; 6 equal arcs.
8. To draw an inscribed regular hexagon; equilateral triangle; square.
9. To make copies of simple designs.
10. To solve problems by means of the circumference formula.
11. To represent graphically the relation $c = \pi d$.
12. The following theorems should be known:

a. Two triangles are congruent if three sides of one are respectively equal to three sides of the other.

b. The circumference of a circle is equal to π times the diameter, or 2π times the radius, i.e., $c = \pi d$, *or* $c = 2\pi r$.

139. Typical problems and exercises. Pupils should be able to give correct answers to the following questions and problems:

Solve by means of equations:

1. The circumference of a circle is 48 feet. Find the radius.

2. The diameter of a circle is 12 feet. Find the circumference.

3. The diameter of a wheel is 3 feet. Find the number of revolutions it makes in passing over a distance of one mile.

4. Make a graph of the equation $c = \pi d$.

5. Write a paper on one of the following topics:
   a. The use of the circle in construction exercises.
   b. The history of finding the value of π.

CHAPTER VIII

FORMULAS AND EQUATIONS

Literal Numbers Which Change in Value

140. Equations studied in preceding chapters. In the preceding chapters we have used the equation as a tool for solving problems. All of the equations, so far, have been of a simple type and easily solved. Thus, in studying perimeters, we found equations of the form $120 = 6s$. In studying triangles we used equations like $4x + 2x + 5x = 180$ to express the sum of the angles. The equation $x + 3x = 90$ may mean that two angles are complementary, and $m + 5m = 180$ may mean that two angles are supplementary. The acute angles of a right triangle satisfy relations like $a + 6a = 90$. The circumference of a circle is found by means of the formula $c = \pi d$. Similar triangles lead to equations like $\frac{x}{5} = \frac{8}{15}$.

All these illustrations show that one cannot go very far in the study of mathematics without a knowledge of algebra (in which letters are often used for numbers), in particular of equations. However, all the equations above can be simplified, as may be seen from the table below.

In each case show how the first form is reduced to the simplified form.

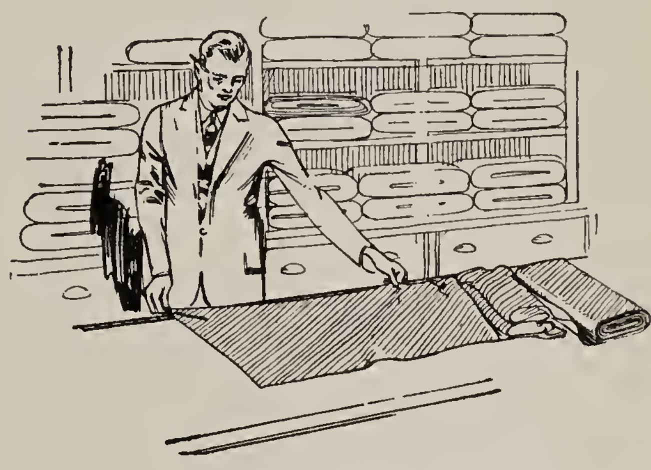
Given Form	*Simplified Form*
$120=6s$	$6s=120$
$4x+2x+5x=180$	$11x=180$
$x+3x=90$	$4x=90$
$m+5m=180$	$6m=180$
$a+6a=90$	$7a=90$
$c=\pi d$	$3.14d=c$
$\frac{x}{5}=\frac{8}{15}$	$15x=40$

In the simplified form all the equations above are of one and the same type. Each may be solved by dividing both members of the equation by the coefficient of the unknown number. It is the aim of this chapter to make us more familiar with equations of this form by bringing in problems from fields other than mathematics; and to extend our knowledge of equations to others which reduce to the same type form, such as $4x+8=20$, or $3x-2=13$.

141. The law of direct variation. We have seen (§137) that in the formula $c=\pi d$ the value of c depends upon the value of d. For every value of d, there is a corresponding value of c. *If d is made to vary* (change) *c varies also, but the ratio $\frac{c}{d}$ remains the same.* Stating this relation between c and d in words, we say that *the circumference varies directly as the diameter*, or that it is *directly proportional* to the diameter.

The following is an example of direct variation. If a yard of cloth sells at \$3, 2 yards sell at \$6, 3 yards

at \$9, etc. Thus, as the number of yards is changed, the price paid changes also. Either *depends* on the other, *i.e.*, to a given number of yards corresponds a certain price and to a given price corresponds a certain number of yards. Moreover, the dependence is such that by *doubling* the number of yards, the price is *doubled*. If the number of yards



is *trebled*, the price is *trebled*, etc. The number of yards and the price paid are said to be *directly proportional* to each other. One is said to *vary directly* as the other.

This type of variation is very common. For example, the more workmen a factory employs, the greater is the pay roll; the time a train travels at a uniform rate varies as the distance; a man's pay depends on the number of days he works. Examples of direct variations are to be studied in this chapter.

Formula for Uniform Motion

142. How to represent the formula graphically. We have seen that the formula for uniform motion is $d=rt$, where d is the number of units (inches, feet, meters) of distance, r the rate, and t the number of units (seconds, hours, days) of time. If a train travels at the rate of 30 miles an hour, the number of miles is 30 times the number of hours, *i.e.*, $d=30t$. As t varies, d also varies;

but the ratio $\frac{d}{t}$ is constant and equal to 30. Hence we may say that d *varies directly* as t (§141).

The relation $d=30t$ may be represented graphically as follows: First make a table of several pairs of corresponding values of d and t (Fig. 265). Thus, let $t=1$, then $d=30$; if $t=2$, then $d=60$; etc.

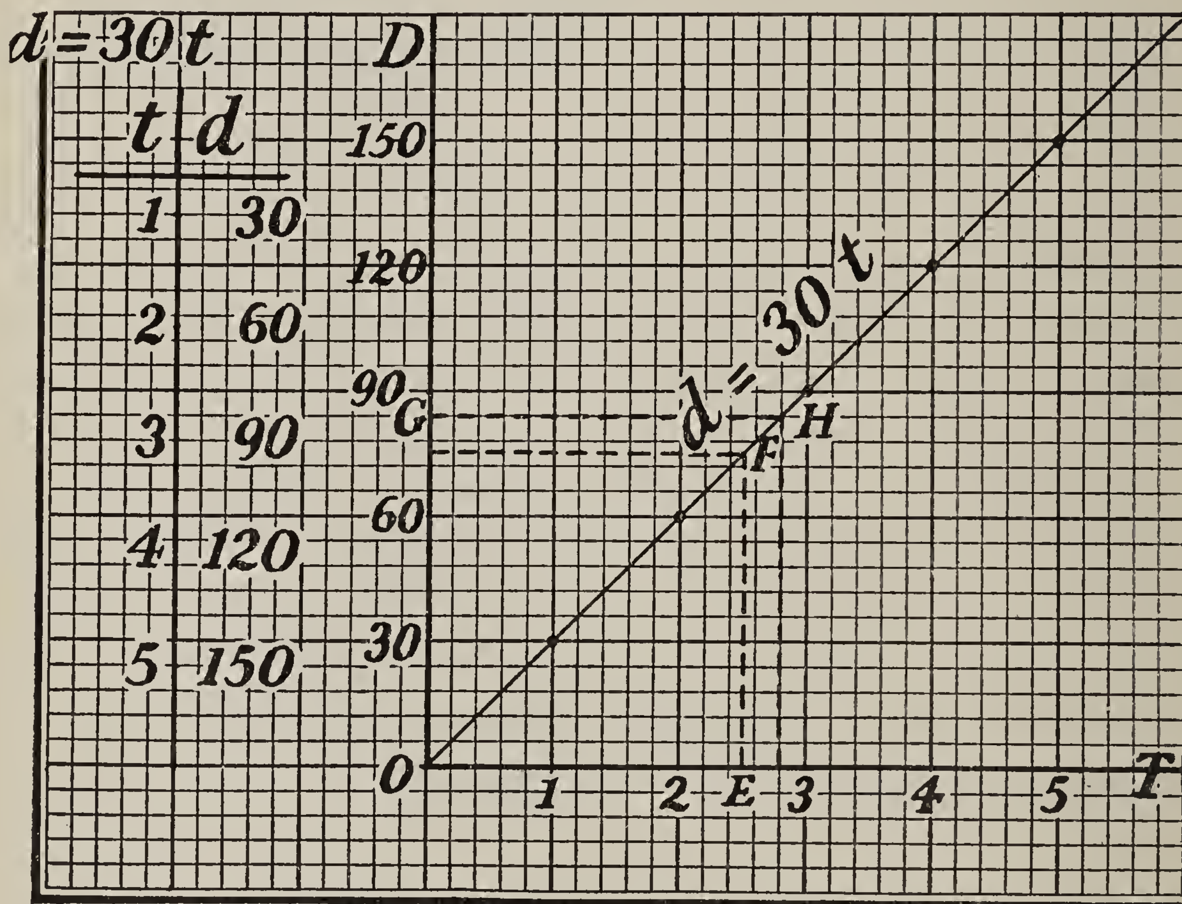


FIG. 265

The next step is to draw the two reference axes, OT and OD.

Convenient units are now chosen for representing graphically the number pairs in the table. These units are marked off on the axes and the points corresponding to number pairs are marked down, or *plotted*. Thus, to

plot the pair (1, 30) pass from O one unit to the right, and then up 30 units. To plot (2, 60) pass from O two units to the right and then up 60 units.

Finally, the line passing through the points is drawn. This is the required graph.

The graph may be used to find values of d for given values of t; or to find t when d is given. Thus, when $t=2\frac{1}{2}$, pass from O two and one-half units to the right to E. From E pass upward to F. This vertical height may now be read off on the line OD. The result is $d=75$.

To find t when $d=85$, pass from O upward along the line OD a distance of 85 units to G. Then pass to the right to the graph to H, and from H pass downward to the line OT. The result is $t=2.8$.

Exercises

1. Make a graph of the equation $d=20t$, following the directions given above.

2. From the graph of Exercise 1, find the values of d corresponding to the values $t=1\frac{1}{2}$, $3\frac{1}{4}$, $2\frac{3}{4}$; find t when $d=40$, 64, 98.

3. Make a graph of the equation $d=18.5t$.

Using the formula $d=rt$, *i.e.*, "Distance is equal to rate times the time," solve the following problems:

4. The speed of a train is 28 miles an hour. If the train leaves the station at 2 P.M., how far from the station will it be at 3:00 P.M.; 3:30 P.M.; 4:20 P.M.; 5:00 P.M.?

5. Sound travels about 1080 feet per second. How far away is a stroke of lightning if the thunder clap is heard 30 seconds after the flash; 20 seconds after the flash?

6. From the newspaper clipping at the right, find out how fast the winner could run. (1:16:30 means "one hour, sixteen minutes, thirty seconds.")

SETHKIEWICZ LOPES TO TRIUMPH IN RACE OVER 15 MILE COURSE

John R. Sethkiewicz, Illinois A. C., won the fifteen mile distance run of the Michigan-Irving C. C. yesterday. The victor covered the course in 1:16:30, finishing well ahead of Frank Stehlak of Palmer Park, who was second.

McHALE WINS AUTO RACE

Kalamazoo, Mich., Aug. 6.—[Special.]—Bud E. McHale of Detroit, driving a Ford special, won the 100-mile auto race here this afternoon. Benny Lawell of Columbus, O., finished second. Time 1:33:31.

7. From the news item at the right, determine how many miles per hour Lawell of Columbus was traveling in his car.

Percentage Formula

143. Rate, or per cent. In a written test a pupil made a score of 22. The total possible score was 25. How may his achievement be measured?

Solution: As a measure of the pupil's achievement of 22 scores in a possible score of 25, we may use the ratio $\frac{22}{25}$.

Dividing 22 by 25, we have $\frac{22}{25} = .88 = \frac{88}{100}$.

This means that a score of 22 out of a possible score of 25 is equivalent to a score of 88 out of a possible 100. The number of "hundredths," which in this case is 88, is the "grade" of the pupil in the test.

Grades are often expressed as "hundredths," because it is then easy to compare the pupil's achievements in several tests. For example, if the same pupil in the next test has a score of 18 out of 20, his achievement $\frac{18}{20}$ is equal to $\frac{90}{100}$ when expressed in hundredths. This is higher than the $\frac{88}{100}$ in the previous test.

The *number of hundredths* is the number of *per cents*

(§19). Thus we say that the pupil's achievements of 22 scores out of 25, or of 18 out of 20, are respectively 88 per cent and 90 per cent. The symbol for per cent is % (§19). With this symbol "88 per cent" is written 88%.

The number of per cent, 88, is called *rate;* $\frac{88}{100}$ is the *rate per cent.*

EXERCISES

1. In the show window of a furniture company the following electric sign is displayed:

What does the sign mean?

2. A clothing firm advertises for a special sale a reduction of 40% on all suits and overcoats. What will be the reduction on a suit which is usually sold for $38?

3. The following poster was displayed in the waiting room of a railroad station:

WHAT HAS HAPPENED TO THE
RAILROADS SINCE 1916?

Increase in revenue	60%
Increase in expenses	110%

What is your answer to the question?

144. To change per cent to a decimal fraction. We have seen (p. 31) that in everyday life measures are frequently expressed as per cent, and that *per cent* means *in hundredths.* In other words, 88 per cent is the same as $\frac{88}{100}$, or as .88. This shows that *per cent may always be expressed as a decimal by dividing the number of per cent by* 100.

Exercises

Express the following as decimals:

1. 24%.

Solution: $24\% = 24 \div 100 = .24$.

2. 5%.	**7.** 48%.	**12.** 415%.	**17.** $4\frac{2}{3}$%.
3. 14%.	**8.** 95%.	**13.** 126%.	**18.** .6%.
4. 20%.	**9.** 263%.	**14.** .7%.	**19.** .07%.
5. 37%.	**10.** 84%.	**15.** $2\frac{1}{2}$%.	**20.** $\frac{1}{8}$%.
6. 22%.	**11.** 321%.	**16.** 4.2%.	**21.** 3.9%

145. To express per cent as a common fraction. We have seen that 88% means the same as $\frac{88}{100}$.

Changing the fraction $\frac{88}{100}$ to lowest terms, we have

$$88\% = \frac{\overset{22}{\cancel{88}}}{\underset{25}{\cancel{100}}} = \frac{22}{25}$$

$$\therefore 88\% = \frac{22}{25}.$$

Thus, *per cent may be expressed as a common fraction by dividing the number of per cent by 100 and then reducing the resulting fraction to lowest terms.*

Exercises

Express per cents as common fractions to lowest terms, as shown above:

1. 11.5%.

Solution: $11.5\% = \frac{11.5}{100} = \frac{\overset{23}{\cancel{115}}}{\underset{200}{\cancel{1000}}} = \frac{23}{200}.$

2. 12%.	6. 45%.	10. $17\frac{2}{3}\%$.
3. 8%.	7. 30%.	11. $37\frac{1}{2}\%$.
4. 16%.	8. $3\frac{1}{2}\%$.	12. $11\frac{1}{9}\%$.
5. 75%.	9. $33\frac{1}{3}\%$.	13. $1\frac{3}{4}\%$.

146. Summary of important per cents expressed as decimals and as common fractions. Certain per cents are so commonly used that they should be memorized.

Write into your notebook a complete table like the one below:

$1\% = .01 = \frac{1}{100}$	$6\frac{2}{3}\% =$	$25\% =$	$62\frac{1}{2}\% =$
$2\% = .02 = \frac{1}{50}$	$8\frac{1}{3}\% =$	$33\frac{1}{3}\% =$	$66\frac{2}{3}\% =$
$3\frac{1}{3}\% = \quad =$	$10\% =$	$37\frac{1}{2}\% =$	$75\% =$
$4\% = \quad =$	$12\frac{1}{2}\% =$	$40\% =$	$80\% =$
$5\% = \quad =$	$16\frac{2}{3}\% =$	$50\% =$	$83\frac{1}{3}\% =$
$6\frac{1}{4}\% = \quad =$	$20\% =$	$60\% =$	$87\frac{1}{2}\% =$

147. Percentage. Base. We have seen that 75% of a number means .75 of it, or $\frac{3}{4}$ of it. Hence, 75% of 360 means $.75 \times 360$, or $\frac{3}{4}$ of 360.

The 360 in this example is the *base*, and $.75 \times 360$ is the *percentage*. In other words, the number of which a per cent is taken is called the **base,** and the result obtained by taking a per cent of a number is called the **percentage.**

If the percentage is denoted by p, the number of per cent by r, and the base by b, the statement *percentage is equal to the rate per cent multiplied by the base* takes the simple form

$$p = \frac{r}{100} \times b.$$

This is the **percentage formula.** It may be used to find any one of the three literal numbers p, r, and b, if the other two are known.

Exercises

Using the percentage formula, find the percentage p in each of the following exercises and problems as shown in the solutions of Exercise 1.

1. 25% of 48.

Since $r=25$ and $b=48$, we may substitute these values in the formula

$$p=\frac{r}{100}\times b.$$

Solution 1: This gives the percentage

$$p=\frac{25}{100}\times 48=\frac{\overset{12}{\cancel{25}\times\cancel{48}}}{\underset{4}{\cancel{100}}}=12.$$

Solution 2: $p=\frac{1}{4}\times 48=12.$

Solution 3: $p=.25\times 48=12.$

2. 5% of 220.

3. $6\frac{2}{3}$% of 90.

4. $6\frac{1}{4}$% of 72.

5. $8\frac{1}{3}$% of 363.

6. 50% of 70.

7. 4% of 29.

8. $33\frac{1}{3}$% of 69.

9. $37\frac{1}{2}$% of 88.

10. $12\frac{1}{2}$% of 48.

11. 75% of 84.

12. $66\frac{2}{3}$% of 66.

13. $16\frac{2}{3}$% of 72.

14. 50% of 92.

15. 7% of 8516.

16. 15% of 673.

17. $3\frac{1}{2}$% of 2500.

18. 125% of 5276.

19. 60% of 327.

20. In a school of 560 pupils, 50% are boys. How many boys are there?

21. A man saves $16\frac{2}{3}$% of his income. If his income is $2400, how much does he save?

22. A girl earning $18 a week plans to use 20% of her income for clothing. How much money must she set aside for clothing during one year?

23. Many families plan in advance the expenditures for each month. A certain amount is set aside for rent, another

for food, another for clothing, etc. Such a proposed plan is called a *budget.* Under ordinary conditions the expenditures may be divided as follows: For food, 25% of the income; for rent, 20%; for clothing, 15%; for current expenses, as coal, gas, light, laundry, car fare, 15%; for recreation, books and papers, church, education, doctor, dentist, insurance, and savings, 25%.

Make out a budget showing how much money a family can spend for the various items above, if the income is $3600. Tabulate the amounts as shown below:

Income	*Food*	*Rent*	*Clothing*	*Current Expenses*	*Health, Recreation, Etc.*

24. A man bought a house for $6200 and sold it with a gain of 15% on the purchase price. How much did he gain?

25. An automobile purchased for $1650 is sold at a reduction of 25%. What was the selling price?

26. A manufacturer announces a 10% bonus for all employees. If a man's salary is $2200, how large a bonus is he going to receive?

27. During a certain year rents increased 20%. If a flat rented for \$60 a month before the increase, for how much did it rent after the increase?

28. The metal of a watch case weighing 2 ounces was 50% gold. How much gold did it contain?

29. During a January sale, the price of \$48 overcoats was reduced $33\frac{1}{3}$%. Find the selling price.

30. A farmer uses 40% of his 800-acre farm for wheat, 20% of the remainder for corn, $33\frac{1}{3}$% of the remainder for oats. How much of the farm is not cultivated?

31. If the expense of operating a factory is 40% of the sales, what is the operating expense for a year in which sales amounted to \$680,370?

32. A family has an income of \$1800 a year. If 20% is used for rent, 30% for food, 30% for clothing, and 10% for incidentals, how much is paid for each of these items? How much is saved?

33. A family with a \$2000 income spent 25% of it for food, 20% for rent, 15% for clothing, and 25% for pleasure. They saved 10% and used the remainder for incidentals. How much was spent for each purpose?

34. A man owns property valued at \$8250. What is the amount of his taxes if the tax rate is \$1.52 per \$100?

35. If a house is insured for \$5000 at an insurance rate of .4%, what is the annual premium?

36. The value of food depends upon the energy it produces. Energy is measured in *calories* of heat. Thus a pound of fat produces 4082 calories of heat, and a pound of protein produces 1814 calories. How many calories of heat are furnished by a pound of butter if it contains 85% fat and 1% protein?

37. A real-estate dealer sold one house at \$6500 and another at \$4300. On the first he made a profit of 12% of the selling price, on the other he lost 10% of the selling price. Find his actual gain or loss.

38. Find the number which increased by 6% of itself gives 424.

Suggestion: Let x be the required number.

Show that $x + .06x = 424$, and solve this equation.

39. A number increased by 10% of itself gives 44. Find the number.

40. Find the number which decreased by 4% of itself gives 192.

41. An article was sold for \$128 at a gain of 10% of the cost. Find the cost.

42. An article was sold for \$175 at a loss of 12% of the cost. What was the cost of the article?

43. A real-estate dealer can sell a certain lot for \$1800. How much can he pay for the lot so as to make 15% on his investment?

44. If fertilizer contains 4% of nitrogen, 10% of phosphoric acid, and 8% of potash, how many pounds of each are there in 1500 pounds of fertilizer?

148. Graphical representation of the percentage formula. If $r = 4$, the percentage formula is $p = .04b$. Notice that this formula is similar to the formula $c = \pi d$, the relation between the circumference and diameter of the circle (§135). We may say that *for a given rate the percentage varies directly as the base.*

The percentage formula may be represented graphically as follows:

1. Let b take the values 0, 50, 100, 150, 200, etc. Tabulate the pairs of corresponding values of b and p.

2. Select convenient units and plot the number pairs in the table.

3. Through the points thus obtained draw a line. This is the graph of the equation $p = .04b$.

b	p
0	0
50	
100	
150	
200	
250	
300	
350	

Exercises

1. Show that the ratio $\frac{p}{b}$ remains constant as p and b vary. Hence p varies directly as b.

2. Make a graph of the equation $p = .05b$.

3. Make a graph of the equation $p = .06b$.

149. Finding the rate by means of the percentage formula. Exercise 1, below, explains how to find what per cent one number is of another.

Exercises

1. What per cent of 33 is 14?

Solution: Let r be the required number of per cent.

By the percentage formula

$$\frac{r}{100} \times 33 = 14.$$

Multiplying by 100,

$$r \times 33 = 1400,$$

$$\therefore r = \frac{1400}{33} = 42\tfrac{14}{33}$$

$$\therefore r\% = 42\tfrac{14}{33}\%.$$

This shows that 14 is $42\frac{14}{33}\%$ of 33, or 42% of 33 approximately. Briefly, the problem may be solved by dividing 14 by 33.

2. Out of 25 games played by a ball team, 18 were won. What per cent was won? What per cent was lost?

3. A pupil solved correctly 9 out of 15 problems. What per cent did he solve correctly?

4. An alloy consists of 18 parts of silver and 5 parts of copper. What per cent of it is silver? What per cent of it is copper?

5. A girl earned \$63.80 during her summer vacation and deposited \$25.00 of this sum in her savings account. What per cent of her earnings did she save?

6. Out of 240 eggs placed in an incubator, 185 chicks were hatched. What per cent of the eggs hatched?

7. On a rainy day 362 pupils were present in a school whose enrollment is 420. What was the per cent of attendance?

8. In a school of 430 pupils 250 are boys. What per cent of pupils are boys?

9. A farmer set out 225 trees; 58 of them died. What per cent died?

10. One out of every three inhabitants in a certain city is foreign-born. What per cent is foreign born?

11. A man pays \$130 for taxes on property valued at \$10,000. What is the tax rate?

12. A dealer gains a profit of $2260 on an investment of $18,000. What is his rate of profit?

13. A library circulated 256,394 books during a certain year. Of these books 162,342 were fiction. What per cent was this of the total number of books?

14. A salesman received $80 for making sales amounting to $3250. What was the rate?

15. The number of books in a library was increased from 930 volumes to 1210. What was the per cent of increase?

Solution: Let r be the number of per cent in the increase.

The actual increase $= 1210 - 930 = 280$.

$$\therefore \frac{r}{100} \times 930 = 280.$$

Dividing both members by 930,

$$\frac{r}{100} = \text{per cent of increase} = \frac{280}{930} = \frac{28}{93}.$$

$$\therefore r = \frac{2800}{93},$$

or $r = 30$ approximately.

Computation:

$$\begin{array}{r} 1210 \\ 930 \\ \hline 280 \end{array}$$

$$\begin{array}{r} 30 \\ 93\overline{)2800} \\ 279 \\ \hline 10 \end{array}$$

16. During two consecutive years the number of immigrants to the United States from Russia increased from 162,395 to 291,040. Find the approximate per cent of increase.

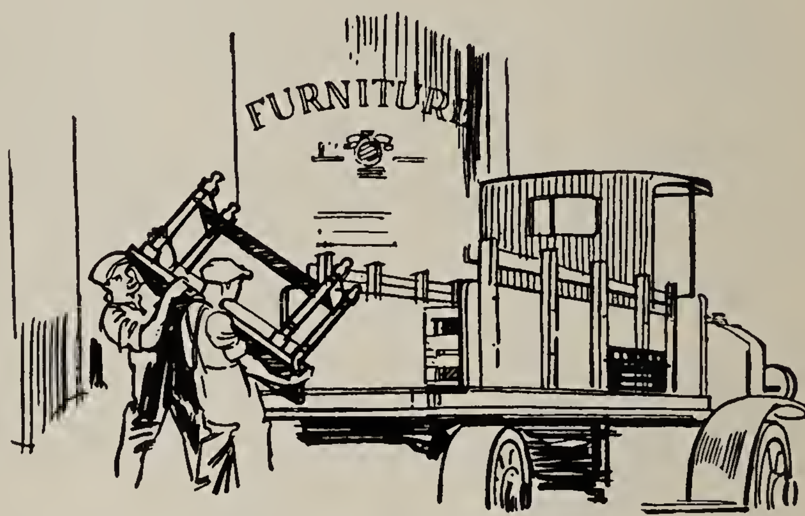
17. A farmer increased his oat crop from 37.5 bushels to 46 bushels per acre. Find the per cent of increase.

18. A man's wages were raised from $4.25 per day to $4.75 per day. Find the per cent of increase.

19. A dealer pays $14.50 for apples and sells them at a loss of $1.50. What is his per cent of loss?

20. An automobile purchased for \$1800 was sold for \$1400 one year later. What was the per cent of the price reduction?

21. Milk was sold for 12 cents a quart in a certain city, and within a few years the price was raised to 14 cents. Find the per cent of increase in the price.



22. A table was purchased for \$16.50 and sold for \$20. The cost of selling, including advertising, packing, delivering, etc., was \$1.30. What per cent of the purchase price was gained? What per cent of the selling price was gained?

23. A man earned \$945 and saved \$56.70 of this amount. What per cent did he save?

24. A pound of crackers sells at 8 cents per pound (16 ounces). The same crackers sell at 5 cents a package weighing $4\frac{1}{2}$ ounces. What per cent does a housekeeper save if she buys crackers in bulk?

25. Due to a safety campaign, the annual number of deaths due to accident was reduced from 54,011 to 51,406. Find the rate of decrease.

26. The average retail price per pound of round steak from 1913–1923 was as follows:

\$.22, \$.233, \$.239, \$.250, \$.296, \$.369, \$.389, \$.395, \$.344, \$.323, \$.355.

Find the per cent of increase for each year.

Tabulate the results as follows:

Year..................					
Price per pound.......					
% increase...........					

150. How to find the base by using the percentage formula. Exercise 1, below, illustrates how the percentage formula may be used to find the base.

Exercises

1. Find the number 5% of which is 75.

Solution:

$$p=75$$
$$r=5$$
$$75=\frac{5\times b}{100}$$
$$7500=5b$$
$$b=\frac{7500}{5}$$
$$\therefore b=1500.$$

2. When gas is made from coal, about 70% of the coal used is changed to coke. How many tons of coal must be burned to make 4500 tons of coke?

3. How much money must a man invest so that a profit of $2550 be 15% of his investment?

4. If only 60% of an army is actually fighting, the others being needed to take care of the fighting men, how large an army is needed if 1,500,000 men are to be available for fighting?

5. In buying a house a man agreed to pay down 25 per cent of the purchase price. This amounted to \$2125. What was the price of the house?

DISCOUNT

151. Meaning of discount. Merchants use percentage in sales of goods. During a January sale a merchant advertised overcoats for sale at a reduction of 20% of the marked price. Such a reduction is called a **discount.** Discounts are often given to attract trade and can be found advertised in most newspapers.

EXERCISES

1. Find the sale price for each of the articles in the adjoining advertisement, using the formula

Sale price = old price minus rate per cent times old price.

2. What is the sale price of an automobile tire listed at \$36.50 with a discount of 20%?

3. A merchant buys \$900 worth of goods with a 10% discount for paying cash. How much does he save by paying cash?

4. Shoes marked at \$5.50 are offered for sale at a discount of 15%. At what price are they sold?

5. A merchant offers house furnishings for sale at a discount of 25%. Find the price of the following goods:

A chair marked	$7.25
A dining table marked	$63.50
A sideboard marked	$52.25
A kitchen range marked	$58.00
A rug marked	$84.00

6. Find the price of 100 spools of cotton at $18.34 per thousand at a discount of 40%.

7. Find the sale price of each of the following articles sold at the discount quoted:

Article	*List Price*	*Discount*	*Sale Price*
Shoes	$ 6.25	15%	
Overcoat	28.00	10%	
Hat	2.00	$12\frac{1}{2}$%	
Gloves	1.50	10%	
Cap	2.25	20%	
Muffler	4.75	15%	

8. During a season when business was slow, a merchant announced certain goods marked down in price for a quick sale. Find the price at which each article was to be sold:

Article	*Old Price*	*Reduction*	*New Price*
Straw hat	$2.25	$33\frac{1}{3}$%	
Sweater	3.50	20%	
Shirt	2.50	20%	
Necktie	2.00	15%	
Belt	2.75	$12\frac{1}{2}$%	

9. In the following advertisement, find the per cent of reduction on a \$65.00 suit:

Solution:

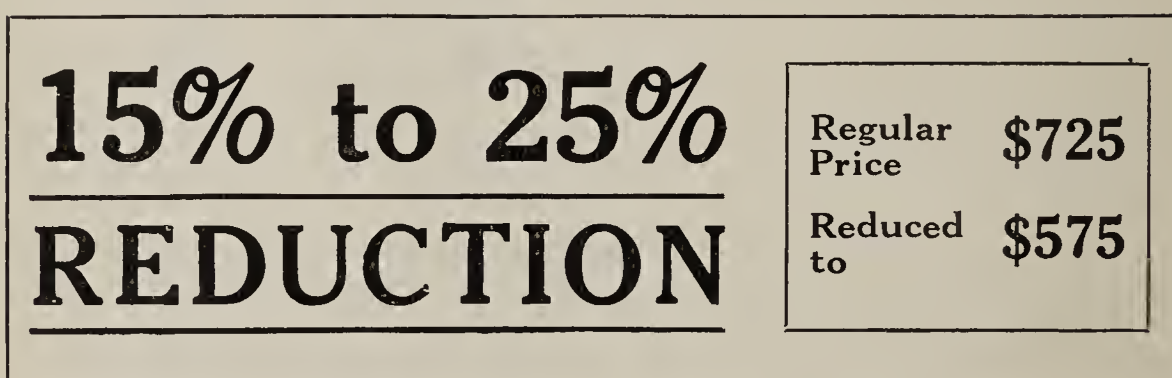
$$\begin{array}{lr} \text{Old price} & =\$65 \\ \text{Sale price} & =\$37.50 \\ \hline \text{Reduction} & =\$27.50 \end{array}$$

By means of the formula we have

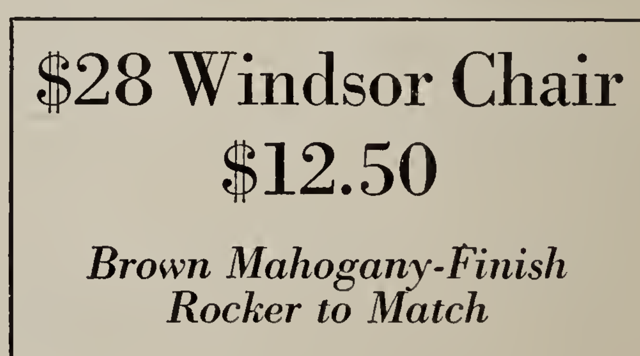
$$27.50=\frac{r}{100}\times 65$$

$$\therefore r=\frac{2750}{65}=42.31$$

10. A firm advertises "Rich furs at tempting prices" at discounts varying from 15% to 25%. Find the reduction on \$725 at 15%; at 25%. Does the discount actually fall between 15% and 25%?



11. The firm selling this chair advertised discounts of 30% to 50%. Is this a better bargain than they advertised?



12. After reading the advertisement, below John's father asked him to find out which was the best bargain as far as the price was concerned.

> **$50**
>
> **$50 SUITS—NOW $35**
>
> ***$65 $75 silk lined suits at $50***

John worked out the answer by mathematics and said that the $75.00 suit was the best bargain. Was he right? Find the per cent of reduction on each suit and tabulate the facts as follows:

Old Price	*Sale Price*	*Reduction*	*%*
$75			
$65			
$50			

13. Are the reductions or discounts in the adjoining advertisement of a uniform rate? Tabulate facts as in Exercise 12.

$\$12^{00}$	**Manhattans,**	$\$8^{35}$
$\$10^{00}$	**Manhattans,**	$\$6^{85}$
$\$8^{00}$	**Manhattans,**	$\$5^{75}$
$\$5^{00}$	**Manhattans,**	$\$3^{45}$
$\$3^{50}$	**Manhattans,**	$\$2^{25}$

14. Are the reductions in the adjoining advertisement uniform?

15. Mary paid \$2.50 for a pair of roller skates before Christmas. After Christmas her sister Jane bought the same make of skates for \$1.78. By what per cent was the price reduced?

16. A dining-room set bought on installments is marked \$145. The first payment is 25% down and the remainder must be paid in 14 equal monthly installments. How much is the first payment, and how much is each of the other payments?

If the set is bought for cash at a discount of 10%, what is the cash price?

17. A wholesale merchant publishes a catalog containing prices of his goods and allowing his customers a discount. In the catalog, the old price is called *list price*, the reduced price after the discount has been subtracted is called *net price*. Thus the net price is equal to the list price diminished by the discount. Find the net price in the table below:

List Price	*Rate of Discount*	*Net Price*
\$ 60	25%	
\$ 48	$33\frac{1}{3}\%$	
\$150	$16\frac{2}{3}\%$	
\$265	20%	
\$345	40%	

18. Many manufacturers issue catalogs with list prices subject to discount for cash payment or payment within a given date. Terms under which discounts are paid are sometimes stated thus: 30 days; 15 days less 2%. This means that bills must be paid in 30 days and that a 2% discount will be allowed if the bill is paid within 15 days. Compute the net amount for the following sale of goods:

4 tennis rackets	at	\$ 3.80
6 pairs of skates	at	\$ 6.10
2 bicycles	at	\$35.50
Total		
Less 2% for cash		
Net		

19. Wholesale dealers and manufacturers often give two or more discounts. These are computed in succession. For example, an article listed at \$565 is subject to discounts of 25% and 20%.

Solution 1: 25% of \$565 = \$141.25

The remainder \$565 − \$141.25 = \$423.75

20% of \$423.75 = \$ 84.75

∴ Net price = \$423.75 − \$84.75 = \$339.

Solution 2: A single discount equal to several successive discounts may be found as follows:

1. Find the sum of the two given discounts, *i.e.*, 25% + 20% = 45%.
2. Subtract the product of the given discounts, *i.e.*, $\frac{45}{100} - \frac{25}{100} \times \frac{20}{100} = \frac{45}{100} - \frac{5}{100} = 40\%$.
3. Hence, the required single discount is 40%.

∴ Net price = \$565 − 40% of \$565 = \$339.

20. Find the net prices in the following list as shown in Solution 1.

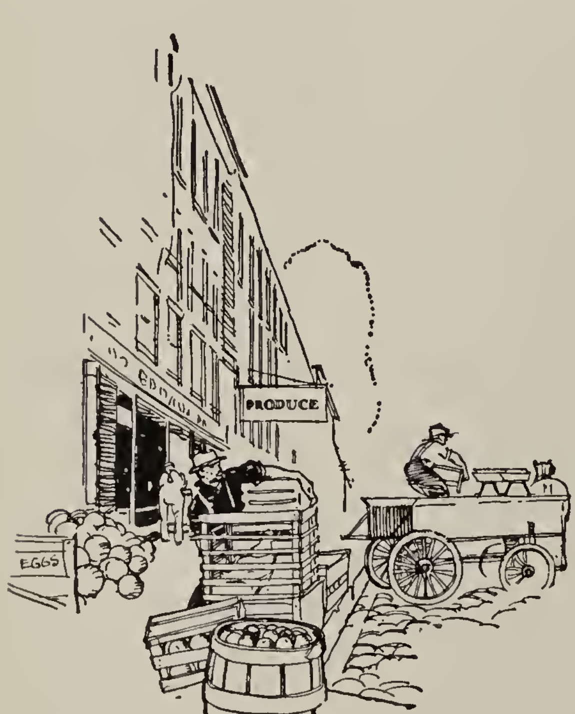
Price List	*Discounts*	*Net Price*
\$18.25	20%, 10%	
22.50	25%, 20%	
34.60	25%, 10%	
42.28	33%, 20%	

21. Change each discount series in Exercise 20 to a single discount and find the net price.

22. Supplies purchased by a millinery establishment amount to \$134.60. Find the net price if a 40% discount is allowed from the list price and 15% from the remainder.

Commission

152. Meaning of commission. A sale is often made by an agent for the producer. For example, a fruit grower ships his fruit to a firm in the city to sell it for him, a manufacturer sends out a traveling salesman to take orders for his goods, and a property owner employs a real-estate agent to sell his property. Payments for such sales may be made in terms of an agreed per cent of the money for which the goods are sold. The amount so paid is called a **commission.**



Exercises

1. A real-estate agent sells a building for \$15,600, receiving a commission of 3%. How much money does he receive?

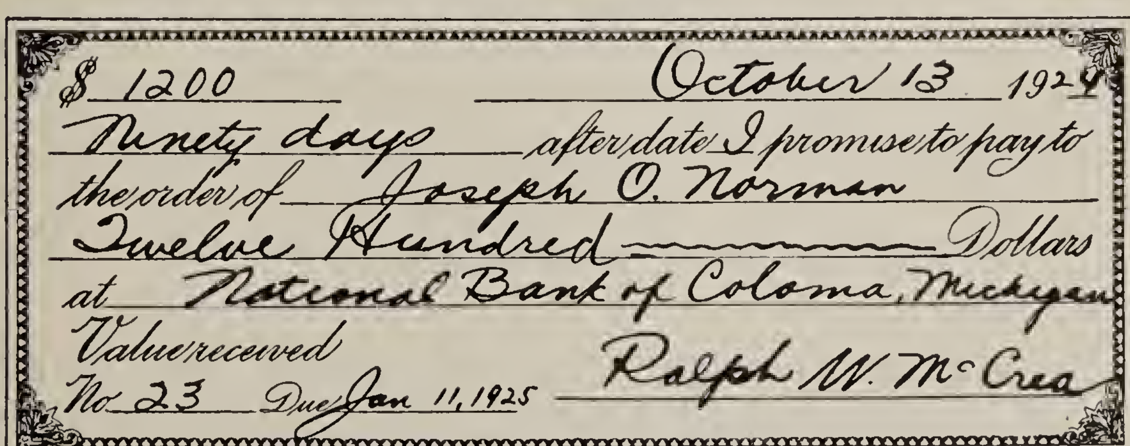
2. A farmer shipped 2,000 bushels of corn to be sold at 55 cents per bushel. If the firm selling the corn charged a commission of 5%, what was the amount received for making the sale?

3. Find the amount of the sales made by an agent who clears \$3500 a year selling on a commission of $1\frac{3}{4}$%.

4. How much money should an agent remit to the owner of a house which the agent sells for \$6250 at a 5% commission?

Interest Formula

153. Interest. Further use of percentage is made in loaning and borrowing money. A man borrows \$500 from a bank. For the use of this money he promises to pay each year 6% of the sum he borrowed. The sum he borrows is the **principal.** The money he pays for the use of the principal is called **interest.** The per cent to be paid is the **rate of interest.**


\$ 1200 October 13 1924
Ninety days after date I promise to pay to
the order of Joseph O. Norman
Twelve Hundred Dollars
at National Bank of Coloma, Michigan
Value received
No. 23 Due Jan 11, 1925 Ralph W. McCrea

A Simple Form of a Promissory Note

The welfare of a person or family depends not only on the income and on the ability to spend the money

wisely, but also on the ability to increase the means of earning money. People who set aside a portion of their income may deposit it in a savings bank for safekeeping. The bank lends the money to others at a sufficiently high rate of interest to be able to pay interest at a lower rate to the depositor. A bank may lend money receiving 6% a year, and pay 3% or $3\frac{1}{2}$% a year for the use of the money. Thus, the money saved earns more money for the depositor.

Exercises

1. Find the interest for 1 year on the following: \$500 at 6%; \$800 at 4%; \$650 at $3\frac{1}{2}$%; \$$p$ at 5%; \$700 at r%; \$$p$ at r%.

2. State the formula for finding the interest for 1 year on p dollars at r%.

3. The interest on \$500 for one year is \$10. Find the rate.

4. A sum invested at $5\frac{1}{2}$% yields interest equal to \$90 a year. How large is the sum?

154. How to use the interest formula in problems. *The interest is equal to the rate per cent times the principal times the number of years.* Denote the interest by i, the rate by r, the principal by p, the time by t, and state the interest formula.

Exercises

1. Find the interest on \$300 at 6% for 3 years.

Solution: Show that $i=\frac{6}{100}\times 300\times 3$

$$=\frac{6\times 300\times 3}{100}=54$$

$\therefore$ Interest $=$ \$54.

2. Find the interest on \$3850 at 5% for 2 years;
on \$4200 at 7% for 3 years;
on \$68 at 8% for 6 months;
on \$250 at 7% for 1 year, 3 months;
on \$325 at $5\frac{1}{2}$% for 1 year, 6 months.

3. Find the interest on \$2400 at 4% for 4 months 12 days.

Solution: $i=\frac{r}{100}\times p\times t.$ Considering a commercial year equal to 360 days, 4 months $=\frac{4}{12}$ of a year, and 12 days $=\frac{12}{360}$ of a year. Hence $i=\frac{4}{100}\times 2400\times\left(\frac{4}{12}+\frac{12}{360}\right)$

$$=\frac{4\times\overset{\overset{4}{\cancel{24}}}{\cancel{2400}}\times 11}{\underset{5}{\cancel{100}\times\cancel{30}}}=\frac{176}{5}$$

$\therefore i=35.2.$

4. Find the interest on \$250 at 4% for 3 years 2 months 10 days; on \$3500 at $3\frac{1}{2}$% for 2 years 3 months 15 days.

5. What sum of money put at interest at 5% yields an income of $800 in 3 years?

Solution: Show that $x \times \frac{5}{100} \times 3 = 800$.

Solve this equation for x.

6. What sum of money at 4% will yield an income of $500 in 3 years?

7. A sum of money is invested at 5%. In 2 years, after the interest is added to the investment, the total amounts to $3500. Find the principal.

Solution: Show that $x + \frac{5x \times 2}{100} = 3500$.

Solve this equation to determine x.

8. What principal must be invested at 6% in order that one may have $3200 at the end of the third year?

9. A **mortgage** is the offering of property as security for the payment of a loan, which becomes void upon payment. Failure to pay back the loan on the date due gives the holder of the mortgage the right to have the property sold at public auction. From this sale he is paid his full amount, the remainder going to the owner of the property. Any legal property may be mortgaged, *e.g.*, furniture, automobiles, even crops which are planted but not yet grown.

On a house worth $8500 the bank holds a mortgage for $2300 at $5\frac{1}{2}$%, interest to be paid every 6 months (semi-annually). How much must be paid each time?

10. What is the semi-annual payment a man must make to a bank holding a mortgage of $1800 on his house, if 6% interest is charged?

11. Mr. James bought an automobile of Mr. Crane, paying $165 in cash and giving a note for $420 payable one year after date at 6% interest. Find the total cost of the automobile.

155. Graphical representation of the interest formula. The graph of the interest formula is similar to that of the percentage formula. Hence, in making the graph, the suggestions of §148 should be followed.

Exercises

1. Using the same axes, and the arrangement shown in §148, make graphs of the equations $i=(.03)p$; $i=(.05)p$; $i=(.06)p$.

2. From the graph find the yearly interest at 5% of \$75; of \$125; of \$275.

A Study of Equations

156. Uses of the equation. The equation has been used for various purposes. First it was used to express the *equality* of two numbers. Thus if x and y are the measures of two angles (Fig. 266), the equation $x=y$ expresses briefly the statement: "The two angles are equal."

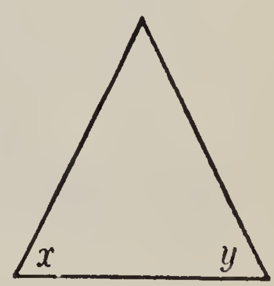


Fig. 266

The equation was also used to express a *relation*, or interdependence, between numbers. For example, if a, b, and c are the measures of the angles of a triangle (Fig. 267), the relation given by the equation $a+b+c=180$ makes it possible to determine one angle if the others are known. Similarly, the equation $c=\pi d$ expresses the relation between diameter and circumference, and $d=rt$ states a relation between time, distance, and rate.

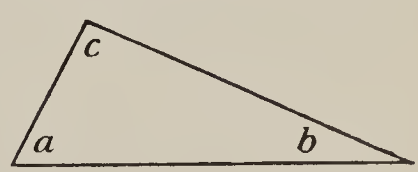


Fig. 267

Furthermore, the equation is used as a brief *statement of a problem.* The statement,

"If a number is increased by 4 times itself, the result is 60," is written briefly $x+4x=60$. The solution of this equation determines the unknown number.

Because equations are important we must learn to solve them and to understand the laws used in the process of solving. For example, we know that the laws known as the multiplication and division axioms enable us to solve such equations as $5x=20$ and $\frac{x}{2}=5$. In the first, we divide each member by 5 to find the required value of x; in the second, we multiply each member by 2.

157. Use of the subtraction axiom in solving equations. *An equation may be regarded as expressing balance between two numbers.* It may be compared with a pair of scales (Fig. 268). If a bag of unknown weight w together with a 5-lb. weight just balances a 10-lb. and a 5-lb. weight, we may express this fact by means of the equation

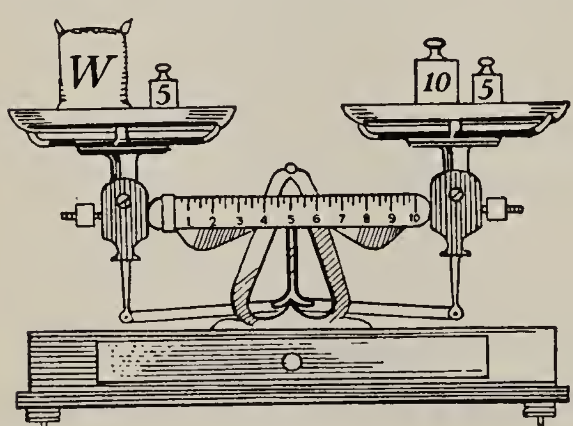


FIG. 268

$$w+5=15.$$

If 5 lb. are taken from each pan, the bag in one pan just balances 10 lb. in the other. In the form of an equation this may be written

$$w=10.$$

Just as a 5-lb. weight may be taken from each pan without destroying the balance, the number 5 may be subtracted from both members of the equation

without destroying the equality. This principle may be stated as follows:

If the same number, or equal numbers, be subtracted from equal numbers, the remainders are equal. This law is the **subtraction axiom.**

The solution of the equation $w+5=15$ may now be arranged in the following brief form:

$$\begin{array}{lr} & w+5=15 \\ \text{Subtracting 5 from both members} & \underline{\quad 5=\ 5} \\ \text{we have} & w\qquad=10 \end{array}$$

Check: Left side:		Right side:
$10+5$		15
15	$=$	15

Exercises

Solve the following equations, arranging your work as shown in the problem above. Check each solution.

1. $w+62=94$.
2. $w+45=80$.
3. $8+h=20$.
4. $100+k=125$.
5. $4.32=s+2.64$.
6. $30=x+10$.
7. $75=23+x$.
8. $1.2=s+.7$.
9. $40=10+t$.
10. $3.72=x+1.86$.

Solve the following problems by means of equations:

11. What number added to 82 will give 157?

12. Twice a certain number increased by 24 is 54. What is the number?

Solution: Let x be the required number.

$$\begin{array}{lr} \text{Then} & 2x+24=54. \\ \text{Subtracting 24 from both sides} & \underline{\quad 24=24} \\ \text{we have} & 2x\qquad=30 \\ \text{Dividing both sides by 2,} & x\qquad=15. \end{array}$$

13. The sum of two angles is 48°. If one angle is 32° larger than the other, how large is each?

14. A sum of $132 is to be divided so that the larger part is 18 greater than the smaller. Find the two parts.

15. A stick 24 inches long is to be cut into two parts so that one is 6 inches longer than the other. Find the length of each part.

Solve the following equations:

16. $6x+2=44$.

17. $2\frac{1}{2}x+7=32$.

18. $7x+4=32$.

19. $24=5x+4$.

20. $38=2+6x$.

21. $15=3b+3$.

Solve the following problems:

22. I am thinking of a certain number. If I treble it and add 6, the resulting number is 30. What is the number?

23. A man has 110 yards of fence with which to inclose a rectangular garden. What are to be the lengths of the sides of the garden if it is to be 24 yards longer than wide?

24. Find two consecutive numbers whose sum is 113.

Suggestion: Consecutive numbers are whole numbers which differ by 1, as 3 and 4, 5 and 6.
Let x be one of the numbers.
Then $x+1$ is the other.

25. Find two consecutive numbers whose sum is 67.

26. Find three consecutive numbers whose sum is 351.

27. Find two consecutive odd numbers whose sum is 68.

Suggestion: Let x be one of the odd numbers.
Then $x+2$ is the other.

28. Find two consecutive odd numbers whose sum is 488.

29. Find two consecutive even numbers whose sum is 230.

30. One angle of a triangle is 32°. The second is 18° larger than the third. How large is each of the three angles?

Suggestion: Let x be the number of degrees in the third angle (Fig. 269).

Then $x+18$ is the number of degrees in the second angle.

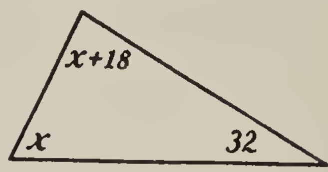


Fig. 269

Hence $x+x+18+32=180$.

Combining the similar terms, we have

$$2x+50=180.$$

In problems 31 to 35 make a sketch before solving:

31. Find the angles of a triangle if the first is 20° greater than the second and the third is 16° less than the second.

32. Find the angles of a triangle if the first is 25° larger than the second and the third is 3 times the second.

33. One of two supplementary angles is 48° larger than the other. Find the angles.

34. One of two complementary angles is 32° larger than the other. Find the angles.

35. The length of a room is 2 feet more than twice the width. The length is 18 feet. Find the width.

36. A boy begins to work in an office with the agreement that he will receive \$35 the first month with an increase of \$3 each successive month, until a maximum of \$65 is reached. In how many months will he be getting \$65?

37. Mary and her brother John raised vegetables to sell to the neighbors. Their sales amounted to \$36. Mary, having spent only half as much time as John, was to get half as much money as her brother. How is the money to be divided?

158. Use of the addition axiom in solving equations. If one of two complementary angles is 40° less than the other, the two angles may be found as follows:

Solution: Let x be the number of degrees in the larger angle.

Then $x-40$ is the number of degrees in the smaller angle.

Since the angles are complementary, we have

$$x+x-40=90$$
$$2x-40=90$$

If we *add* 40 to both members of this equation we have

$$2x-40+40=90+40$$
$$\text{or, } 2x+40-40=130$$
$$\therefore 2x=130$$
$$\text{and } x=65$$
$$x-40=25.$$

Briefly, this solution may be arranged in the following form:

$$
\begin{array}{lr}
 & x+x-40 = 90 \\
\text{Adding 40 to both} & \\
\text{members of the equation} & 40 = 40 \\
\hline
\text{we have} & 2x = 130 \\
\text{and} & x = 65.
\end{array}
$$

When adding 40 to both members of the equation, we are using the following law:

If the same number, or equal numbers, be added to equal numbers, the sums are equal. This is known as the **addition axiom.**

Exercises

In each of the following equations find the exact value of the unknown, and check the results. Arrange the work as shown in the problem above.

1. $x-3=8$.
2. $x-10=5$.
3. $3y-4=8$.
4. $7y-2=12$.
5. $2k-7=15$.
6. $3.1a-9.1=3.3$.
7. $.2m-14.8=8$.
8. $4.7t-6.8=7.3$.
9. $1.6x-12.8=5.5$.
10. $.6x-4.1=3.1$.

Solve the following problems by means of equations:

11. A boy is 4 years younger than his brother. The sum of their ages is 16. Find the age of each.

12. One of two supplementary angles is 20° less than the other. Make a sketch of the two angles and find them.

13. If 5 times a number is diminished by 10 the result is 35. Find the number.

14. The perimeter of a triangle is 118 feet. One side is 5 feet shorter than the second, and the third is 12 feet longer than the second. Find the three sides.

15. One of two complementary angles is 22° smaller than the other. Find the two angles.

16. If 4 times a number is decreased by 5, the result is 55. Find the number.

17. A sum of \$20 was divided between two persons. One received \$4 less than the other. How much did each receive?

18. The width of a basketball court is 20 feet less than the length. The perimeter is 240 feet. Find the dimensions.

19. Two opposite angles are denoted respectively by $3x+12$ and $5x-8$. Find the angles.

Suggestion: Make a sketch. Form the equation. Add 8 to both members. Subtract $3x$ from both.

20. John has 6 marbles more than Henry. Together they have 26 marbles. How many marbles has John?

21. A man who paid \$8000 for his house wishes to sell it with a profit of \$500. How much must he ask for the house, if he pays a commission of 3% of the selling price to a real-estate agent for making the sale?

22. A boy had 3 times as much money as his brother. Having spent 22 cents he had as much as his brother. How much money did he have?

Solve the following equations and check each result:

23. $6x-6=126-5x$.

Suggestion: Add $5x$ to both sides; then add 6 to both sides.

24. $5x-7=2x+5$.

25. $15x+10=150-5x$.

26. $4m+6=9m-14$.

27. $2r+17=5r+5$.

28. $8a-10=6a+20$.

29. $6b+14=3b+17$.

30. Ellen is 4 years younger than Mary. The sum of their ages is 20 years. How old is each?

159. Use of the multiplication axiom in solving fractional equations. The solution of equations containing fractions can often be simplified by using the multiplication axiom stated in §88. Thus by multiplying every term of the equation $\frac{t}{5}+\frac{t}{3}=16$ by the *least*

common multiple of the denominators, we get the equation

$$\frac{\overset{3}{\cancel{15}}t}{\cancel{5}}+\frac{\overset{5}{\cancel{15}}t}{\cancel{3}}=15\times 16$$

Reducing the fractions in this equation, we find that $3t+5t=240.$

By combining similar terms we have $8t=240.$

Dividing both members of this equation by 8, $t=\ 30.$

This result may be *checked* by substituting 15 for t in the *original* equation. This may be done as follows:

Check: Left side: Right side:

$$\begin{array}{rcr} \frac{30}{5}+\frac{30}{3} & & 16 \\ 6+10 & & 16 \\ 16 & = & 16 \end{array}$$

Exercises

Solve the following equations and check the results:

1. $\frac{x}{4}+\frac{x}{7}=22.$

2. $\frac{y}{3}-\frac{y}{6}=10.$

3. $\frac{2r}{3}+\frac{3r}{4}=23.$

4. $\frac{9x}{2}-\frac{5x}{3}=28.$

5. $\frac{3t}{4}-\frac{4t}{7}=5.$

6. $\frac{x}{3}+\frac{x}{6}-\frac{x}{4}=18.$

7. $3.4y-1.2y+4.8y=70.$

8. $8x-4.5x+5.2x=870.$

9. $20.2x-15.2x+.6x=28.$

160. What every pupil should know and be able to do. At the end of this chapter the pupil is expected to be able:

1. To change per cents to decimal fractions and to common fractions.

2. To solve problems in percentage and interest by means of formulas.

3. To make graphs of equations of the form $ax=b$, a and b being whole numbers, decimal or common fractions.

4. To solve equations of the forms

$w+5=20$; $5w+3=18$; $2x-40=90$; $8a-10=6a+20$; $\frac{9x}{2}-\frac{5x}{3}=28$.

5. The following principles and formulas should be known:

a. If equal numbers are added to equal numbers, the sums are equal. (Addition axiom.)

b. If equal numbers are subtracted from equal numbers, the remainders are equal. (Subtraction axiom.)

c. The percentage formula: $p=\frac{rb}{100}$

d. The interest formula: $i=\frac{rpt}{100}$

e. The law of uniform motion: $d=rt$

161. Typical problems and exercises. The following represent types of problems every pupil should be able to solve:

1. The winner of a 100-mile automobile race finished in 95 minutes, 18 seconds. How fast did he travel?

2. A man saved 20% of his income of \$3500. How much did he save?

3. In a school of 480 pupils, 260 are girls. What per cent are girls?

4. In a test a pupil makes a score of 16 out of a possible score of 22. What is his grade?

5. Find the number which increased by 6% of itself gives 424.

6. Find the number 5% of which is 75.

7. Express as a common fraction, and as a decimal fraction, each of the following: 5%, 10%, $12\frac{1}{2}$%, 20%, 30%, 45%, 50%, 75%.

8. One of two complementary angles is 48° smaller than the other. Find the two angles.

9. Which is the better bargain so far as price reduction is concerned, a \$55 suit at \$38, or a \$65 suit at \$48?

10. Find the interest on \$3850 at $6\frac{1}{2}$% for 2 years.

Solve the following equations:

11. $w+60=84$.

12. $125=105+x$.

13. $7y-2=12$.

14. $2x+15=5x+3$.

15. $\frac{3x}{4}+\frac{4x}{7}=5$.

16. $3.4a-1.2a+4.8a=70$.

17. Make a graph of the equation $p=4.6b$.

18. Write a paper on one of the following topics:

a. The value of the algebraic formula.
b. The graph of a formula.
c. The use of mathematics in business.
d. The equation as a tool for solving problems.

CHAPTER IX

SUPPLEMENTARY ARITHMETICAL EXERCISES, PROBLEMS, AND TESTS

Practice Exercises in the Fundamental Operations

162. Why practice is needed. In the first six grades you spent much time training yourself to become accurate and quick in arithmetical work. In the junior high school you must continue practice in arithmetic. Ordinarily this is supplied by the regular classroom work. Whenever you feel that more training is needed, you should practice working the exercises in this chapter, and thereby supplement the regular class work. Like the athlete who practices daily to keep physically fit, you must keep up practice in number work to preserve your skill in arithmetic. Lack of ability to perform arithmetical operations with ease and accuracy interferes seriously with all classroom work.

When working the exercises place a sheet of paper under one line at a time and write your results on it.

163. Exercises in addition. Some seventh-grade pupils do easily six of the following exercises correctly in three and one-half minutes. See how you compare with them.

1.	2.	3.	4.	5.	6.
503	794	692	533	326	679
348	859	510	259	583	752
721	123	684	142	173	606
394	487	340	871	569	128
249	431	321	786	771	392
271	165	659	579	128	780
573	877	789	516	648	500

7.	8.	9.	10.	11.	12.
138	269	337	162	173	529
406	434	457	648	235	966
353	695	500	589	769	686
584	767	428	600	746	725
989	986	364	434	504	631
320	252	434	727	600	589
226	204	754	550	986	132

13.	14.	15.	16.	17.	18.
.003	1.842	2.598	1.18	1.056	10.254
.56	5.83	4.63	9.75	.65	8.59
.49	5.60	2.46	9.366	2.98	66.89
8.59	2.98	8.476	8.06	34.64	297.82
135.86	32.84	17.84	38.29	24.59	26.49

164. Exercises in subtraction. When you can get 10 correct results to these subtraction problems in 3 minutes you are doing as well as most seventh-grade pupils.

1.	2.	3.
780874	840132	460430
158364	99396	125472
4.	**5.**	**6.**
839844	528796	169675
582496	297472	18038
7.	**8.**	**9.**
823832	817018	798916
56198	365112	257258
10.	**11.**	**12.**
112365	178631	243617
105726	108725	217403
13	**14.**	**15.**
389792	461249	648177
145747	231318	572153
16.	**17.**	**18.**
234582	561893	980736
166579	179170	817804
19.	**20.**	**21.**
356112	724042	563091
184821	516983	217582

165. Exercises in multiplication. You should be able to do 5 of these multiplication exercises correctly in $4\frac{1}{2}$ minutes.

1.	2.	3.	4.	5.	6.
5419	7593	2835	2967	8903	1350
482	374	846	738	927	842

7.	8.	9.	10.	11.	12.
1462	7590	4232	6845	1586	7968
325	189	674	543	417	345

13.	14.	15.	16.	17.	18.
42592	3862	8937	2439	7586	4562
865	562	417	512	684	189

166. Exercises in division. Many pupils can do 5 of these division exercises correctly in $4\frac{1}{2}$ minutes. See if you can do better.

1.	2.	3.	4.
23)9568	76)32148	47)17249	89)28569
5.	**6.**	**7.**	**8.**
58)35264	47)298826	96)56064	56)291088
9.	**10.**	**11.**	**12.**
47)27589	37)242239	98)102626	92)80752
13.	**14.**	**15.**	**16.**
92)227056	37)221001	85)628575	64)414848

167. Adding and subtracting fractions. Place a sheet of paper to the right of the equality signs and then add and subtract as indicated. Work the problems in columns.

1. Time: $2\frac{1}{2}$ minutes.

$\frac{1}{2}+\frac{1}{4}=$	$\frac{1}{2}+\frac{3}{4}=$	$\frac{5}{12}+\frac{1}{4}=$
$\frac{1}{2}+\frac{3}{4}=$	$\frac{3}{4}-\frac{3}{8}=$	$\frac{3}{16}+\frac{5}{8}=$
$\frac{1}{4}-\frac{1}{8}=$	$\frac{3}{8}+\frac{1}{4}=$	$\frac{1}{5}-\frac{1}{10}=$
$\frac{1}{3}-\frac{1}{6}=$	$\frac{7}{8}-\frac{1}{4}=$	$\frac{2}{3}+\frac{1}{6}=$
$\frac{1}{2}-\frac{1}{8}=$	$\frac{5}{6}-\frac{1}{2}=$	$\frac{1}{3}-\frac{1}{9}=$
$\frac{1}{2}+\frac{1}{16}=$	$\frac{2}{3}+\frac{1}{6}=$	$\frac{2}{3}-\frac{4}{9}=$

2. Time: 10 in $2\frac{1}{2}$ minutes.

$\frac{1}{2}-\frac{1}{3}=$	$\frac{3}{5}+\frac{5}{6}=$	$\frac{3}{8}+\frac{4}{5}=$
$\frac{1}{3}+\frac{1}{4}=$	$\frac{5}{6}-\frac{5}{8}=$	$6+\frac{3}{8}=$
$\frac{2}{3}-\frac{1}{4}=$	$\frac{4}{5}+\frac{3}{4}=$	$5-\frac{1}{5}=$
$\frac{1}{3}+\frac{3}{4}=$	$\frac{3}{4}+\frac{11}{14}=$	$\frac{2}{3}+\frac{3}{2}=$
$\frac{1}{5}+\frac{1}{3}=$	$\frac{7}{8}-\frac{1}{3}=$	$\frac{9}{16}+\frac{3}{5}=$
$\frac{2}{3}-\frac{1}{5}=$	$\frac{7}{12}-\frac{3}{8}=$	$\frac{5}{6}+\frac{2}{7}=$

3. Time: 5 in 3 minutes.

$8\frac{3}{4}-6\frac{2}{3}=$	$5\frac{1}{4}+6\frac{1}{3}=$	$5\frac{1}{3}+3\frac{3}{5}=$
$9\frac{1}{4}-8\frac{7}{8}=$	$6\frac{3}{8}-2\frac{3}{5}=$	$2\frac{2}{3}+5\frac{3}{4}=$
$9\frac{1}{2}+11\frac{3}{4}=$	$3\frac{4}{5}-2\frac{3}{4}=$	$9\frac{1}{3}-7\frac{4}{5}=$
$6\frac{3}{2}+4\frac{2}{3}=$	$5\frac{3}{7}+2\frac{1}{4}=$	$12\frac{3}{4}-3\frac{5}{6}=$
$14\frac{1}{2}-8\frac{2}{3}=$	$2\frac{3}{4}-1\frac{7}{15}=$	$13\frac{5}{11}+8\frac{3}{5}=$

168. Exercises in multiplying and dividing fractions. Multiply and divide as indicated. Work these exercises in columns.

1. Time: 3 minutes.

$\frac{3}{4} \times 46 =$	$10 \times \frac{7}{15} =$	$\frac{3}{8} \times 6 =$
$\frac{3}{8} \times 120 =$	$8 \times \frac{5}{12} =$	$9 \times \frac{7}{12} =$
$\frac{13}{5} \times 70 =$	$10 \times \frac{3}{16} =$	$\frac{2}{5} \times 6 =$

2. Time: $2\frac{1}{2}$ minutes.

$\frac{1}{2} \times \frac{1}{3} =$	$\frac{4}{9} \times \frac{3}{5} =$	$\frac{5}{9} \div \frac{2}{3} =$
$\frac{5}{8} \div \frac{1}{4} =$	$\frac{7}{8} \div \frac{3}{4} =$	$\frac{8}{9} \times \frac{3}{16} =$
$\frac{5}{9} \div \frac{4}{15} =$	$\frac{2}{3} \times \frac{3}{5} =$	$\frac{4}{5} \times \frac{5}{9} =$

3. Time: 5 minutes.

$4 \times 2\frac{1}{2} =$	$2\frac{3}{4} \times 44 =$	$121 \div 2\frac{3}{4} =$
$6 \times 3\frac{1}{6} =$	$8\frac{5}{6} \times 6 =$	$3 \div 2\frac{1}{5} =$
$53 \div 8\frac{5}{6} =$	$3\frac{1}{3} \times 12 =$	$6\frac{2}{5} \div 16 =$
$67 \div 8\frac{3}{8} =$	$3\frac{2}{5} \times 16 =$	$12 \times 2\frac{3}{4} =$

4. Time: $4\frac{1}{2}$ minutes.

$7\frac{1}{2} \times \frac{3}{4} =$	$5\frac{4}{5} \div \frac{2}{3} =$	$6\frac{1}{2} \times 3\frac{1}{4} =$
$5\frac{1}{2} \times \frac{3}{5} =$	$6\frac{2}{5} \div \frac{4}{5} =$	$2\frac{2}{5} \div 1\frac{3}{5} =$
$1\frac{3}{4} \times 2\frac{1}{3} =$	$4\frac{1}{12} \div 2\frac{1}{3} =$	$6\frac{3}{4} \times 8\frac{4}{5} =$

169. Exercises in multiplying and dividing decimal fractions. Multiply or divide as indicated, working the exercises in columns.

1. Time: 5 minutes.

$$\begin{array}{r} 3.141 \\ \underline{2.6} \end{array} \qquad \begin{array}{r} 8.755 \\ \underline{3.34} \end{array} \qquad \begin{array}{r} 36.4 \\ \underline{52.61} \end{array} \qquad \begin{array}{r} 10.08 \\ \underline{4.59} \end{array}$$

$$\begin{array}{r} .006 \\ \underline{1.49} \end{array} \qquad \begin{array}{r} .674 \\ \underline{5.32} \end{array} \qquad \begin{array}{r} 152.3 \\ \underline{.006} \end{array} \qquad \begin{array}{r} 91.5 \\ \underline{3.02} \end{array}$$

2. Time: $5\frac{1}{2}$ minutes.

$$25\overline{)82.25} \qquad .468\overline{)711.36} \qquad .368\overline{)17.296}$$

$$19\overline{).6213} \qquad 4.16\overline{)956.8} \qquad 260\overline{)32.5}$$

170. Practice exercises in finding percentages. Find the following percentages:

Time: 14 minutes.

6 % of 1240 =	50 % of 378 =
5 % of 217 =	26 % of 280 =
3 % of 120 =	$12\frac{1}{2}$% of 926 =
4 % of 75 =	59 % of 81.4 =
$4\frac{1}{2}$% of 360 =	$37\frac{1}{2}$% of 225 =
$6\frac{1}{2}$% of 312 =	43 % of 169 =
7 % of 1.75 =	$16\frac{2}{3}$% of 1600 =
$5\frac{1}{2}$% of 6.7 =	$33\frac{1}{3}$% of 3200 =
$3\frac{1}{2}$% of .85 =	$87\frac{1}{2}$% of 524 =
8 % of 2400 =	25 % of 388 =
$2\frac{1}{2}$% of 800 =	$66\frac{2}{3}$% of 17.57 =
7 % of 81.4 =	$83\frac{1}{3}$% of 2279 =
$6\frac{1}{4}$% of 6118 =	.1 % of 4365 =

Arithmetical Problems

The problems in §§171 to 178 are to give you practice in solving problems.

171. Problems of the home.

1. The purpose of keeping an account is to have a complete record of all money received and paid out, together with the items for which money has been paid and the sources from which it has been received. Keeping an account helps us to spend money wisely and carefully. The following is a personal account kept by a pupil of the seventh grade for one week.

Find the totals and the balance on hand.

Date		*Receipts*	*Expenditures*
Sept. 3	Allowance received.	$1.25	
"	Paid for lunch.		$0.30
"	Paid for writing material.		.15
"	Paid for candy.		.10
4	Paid for amusement.		.20
"	Earned by going on errands. . . .	.15	
"	Paid for lunch.		.25
5	Earned by carrying out ashes. . .	.25	
"	Paid for lunch.		.24
6	Paid for soda.		.15
"	Paid for car fare.		.14
7	Paid for ticket.		.25
"	Paid for lunch.		.30
"	Earned by helping neighbor. . . .	.50	
8	Earned by working in store. . . .	.50	
9	Paid to Sunday School.		.15
	Totals.		
Sept. 10	Balance on hand.		

2. Make an account sheet; enter the following items and balance the account:

Oct. 1 Received allowance	$2.00	Oct. 5 Received for working	$0.40
2 Paid for soda	.10	6 Paid for car fare	.28
3 Paid for church	.10	7 Paid for magazine	.25
4 Paid for lunch	.23		

3. On a sheet of paper ruled as shown in Exercise 1, keep a personal account of your income and expenses for one week.

4. Some families keep accounts of expenditures. A family of six took a 200-mile automobile trip, and a careful account of all expenses was kept, as follows:

September 6

Gasoline	$ 3.15
Meals	11.75
Amusement	3.60
Incidentals	.80

September 7

Lodging	8.00
Meals	12.20
Incidentals	3.60

September 8

Lodging	6.50
Gasoline	3.57
Oil	1.80
Incidentals	1.65
Meals	11.40

Find the total expense for the trip.

5. A man bought a new automobile for $1650. He traveled 6832 miles during the first year. The cost of keeping it includes the following items:

Depreciation	$250.00
Interest at 6%	
Oil for each 500 miles	3.25
Gasoline, at 21 cents a gallon, 14 miles per gallon	
Four tires $32.50 each	
Insurance	43.75
Repairs	28.50
Garage rent	180.00

What was the total cost of keeping the automobile during the first year?

6. A family having an income of $3600 a year agreed upon the following family budget: 25% for rent, 2% for light and telephone, $2\frac{1}{2}$% for fuel, 30% for food, 20% for clothing, 2% for amusement, 1% for gifts, 15% for the savings account, and the remainder for other expenses. How much is spent for each item?

7. Henry says that he can save 85 cents a week out of his allowance. How long will it take him to have enough money to buy a bicycle costing $32.75?

8. If your gas bill for the last month is $2.48, with 10% off for cash within 10 days, how much does your father save by paying the bill within 10 days?

9. One of two boys employed in the same office earns $25 a week and saves $7.50; the other, earning $22 a week, saves $7.25. What per cent of his salary does each save?

10. A successful housekeeper saves by buying at the right time. By watching the advertisements, a mother bought

the following articles at the discounts indicated. Find the total saving.

Articles Purchased	*Regular Price*	*Discount*	*Sales Price*
2 Boys' overcoats.......	$18.50 each	40%	
1 Girls' suit............	35.00	30%	
1 Pair of shoes..........	9.50	15%	
2 Silk shirts............	6.25 each	35%	
2 Pairs of gloves........	1.75 each	30%	
1 Set of dinner dishes....	28.50	$22\frac{1}{2}$%	

Total saving =

11. If it takes $\frac{3}{4}$ flat tablespoon of butter, $\frac{3}{8}$ of a cup of milk, 1 cup of flour, 2 teaspoons of baking powder, $\frac{1}{4}$ teaspoon of salt, and $\frac{3}{4}$ flat tablespoon of lard to make 6 biscuits, how much of each is to be used for making 10 biscuits?

12. If 350 watts run a washing machine, and if electric current costs 9 cents per kilowatt hour, how much does it cost per hour to wash with the machine?

13. The cost of gas is an item to be considered in cooking. If a gas burner uses 7 cubic feet of gas per hour, with gas costing $.95 per thousand cubic feet, what is the cost for gas in baking a roast $1\frac{3}{4}$ hours?

14. During this season my mother paid $42.80 for ice. She was charged 30 cents per fifty pounds. How much ice did we use?

15. How large a freezer should be used to make $2\frac{3}{4}$ quarts of ice cream, if the freezer should be only 75% full before freezing?

16. A certain brand of coffee sells at 39 cents a pound. The same coffee can be bought in 3-pound cans for $1.10.

What is the saving on 12 pounds by buying it in 3-pound cans?

17. For her birthday party Ruth used the following recipe for making cocoa: To make 4 cups of cocoa, use 2 cups of milk, 2 cups of hot water, $1\frac{1}{2}$ tablespoons of cocoa, and 2 tablespoons of sugar. How much of each did she use to make 9 cups?

18. In canning blackberries 2 cups (1 pint or 1 pound) of sugar are required for 2 quarts of berries. If sugar costs $6.80 per hundred pounds (cwt.), berries $1.90 a crate (24 qt.), and quart jars 65 cents a dozen, what will be the cost of a quart of canned berries?

172. Problems of the store.

1. When eggs are selling at 41 cents a dozen, how much money do you need to buy $2\frac{1}{2}$ dozen?

2. A remnant of silk $\frac{5}{8}$ yard long sold for $2. How much is that for one yard? Another remnant of the same quality $\frac{7}{8}$ yard long sold for $2.20. Which of the two remnants was the better bargain?

3. For the Thanksgiving day dinner John's father bought a $12\frac{1}{2}$-pound turkey at 45 cents a pound. How much did he pay for the turkey?

4. Mary bought a roast of beef weighing $5\frac{3}{4}$ pounds at 28 cents a pound. What was the price paid for the roast?

5. If oranges sell at 45 cents a dozen, how much will 18 oranges cost?

6. Garden hose sells at $2.80 for 25 feet. What is the price of 80 feet?

7. If collars sell at 2 for 25 cents, how much will 5 collars cost?

8. If potatoes are selling for $1.10 a bushel, what is the value of a load weighing 1320 pounds?

9. How many pieces of ribbon $\frac{3}{4}$ yard long can be made from 10 yards?

10. How many towels $\frac{7}{8}$ yard long can be made out of a piece of material $3\frac{3}{4}$ yards long?

173. Finding averages.

1. The following data were taken from the monthly budget of a family of four. Find the average daily expenditure.

Date	*Cost*	*Date*	*Cost*
Oct. 1	$1.93	Oct. 16	$1.12
2	2.26	17	1.95
3	1.63	18	1.64
4	1.48	19	1.48
5	1.72	20	3.42
6	3.31	21	.45
7	.46	22	1.37
8	1.65	23	1.91
9	1.78	24	2.08
10	1.03	25	3.30
11	2.25	26	.20
12	1.57	27	2.96
13	2.05	28	1.12
14	.67	29	1.84
15	1.98	30	1.72
		31	1.65
			31)

Average daily expenditure = $

2. The following scores were made by 22 pupils in a test in arithmetic. Find the average.

25, 24, 24, 22, 20, 20, 20, 19, 18, 18, 17, 16, 14, 14, 13, 12, 11, 11, 11, 8, 8, 7.

3. Find the average height of the boys in a class if their heights, in feet, are as follows:

5.2, 5.0, 5.0, 4.9, 4.8, 4.6, 4.5, 4.4, 4.3

4. The attendance in a junior high school for a week was as follows:

Monday	536	Thursday	528
Tuesday	570	Friday	531
Wednesday	564		

Find the average daily attendance.

5. A dealer bought 70 rugs at $95 each. He sold 52 of them at $130 each, and 18 at $115 each. Find the average profit made.

174. Problems of the farm.

1. If 13.5% of the beets raised by a farmer is sugar, how many pounds of sugar can be made from 643 tons of beets?

2. For 60 head of cattle a farmer used 88 tons of hay and 37 tons of straw during the winter. How much hay and straw will he need for 80 head?

3. A farmer who could have sold his potatoes for $.85 in the fall, kept them until spring and sold them for $1.10 a bushel. If about $\frac{1}{5}$ of the potatoes spoiled during the winter, did he gain or lose by holding them over? What was his loss or gain?

4. If $3\frac{1}{2}\%$ of the weight of a cow's milk is butter fat, how many pounds of butter fat are contained in 32 pounds of milk?

5. A cow gives an average of 15 pounds of milk a day and the milk tests 4.1% of butter fat. How much money is received for butter from the cow in one year, if butter averages 32 cents a pound?

6. If a barrel contains $2\frac{3}{8}$ bushels of apples and sells at $5.50, how much does a farmer receive for 133 bushels of apples?

7. A farmer set out 260 trees, 213 of which grew. What per cent of the trees grew?

8. Mr. Jones shipped a carload of 25 cattle weighing 29,950 pounds. Find the average weight per head?

9. If a horse eats 12 quarts of oats each day, how many bushels of oats are needed to feed a team of 2 horses for a year?

10. At a store, a farmer wishes to trade several pounds of butter for 2 pounds of tea selling at 60 cents a pound, 6 pounds of coffee at 40 cents a pound, 10 pounds of sugar at $8\frac{1}{2}$ cents a pound, and 12 bars of soap at 6 cents a bar. If he asked 37 cents a pound for the butter, how much butter would he have to give in trade?

11. A farmer trades 16 tons of hay for coal. If hay is worth $20 a ton and coal $14.50 a ton how much coal should he receive?

12. A farmer paid $7216 for his 82-acre farm. What was the value per acre?

13. A man bought a 135-acre farm at $80 per acre, one-third cash, the balance remaining at 6% interest. Find the price of the farm, the first payment, and the interest per year on the balance.

14. What is the commission on 620 bushels of potatoes at $1.75 a bushel, if the rate charged is 5%?

15. Mr. Johnson raised 1825 bushels of wheat one year. By fertilizing he increased the crop by $493\frac{1}{2}$ bushels the next year. Find the per cent of increase.

175. Percentage problems.

1. Out of his salary of \$28 a week a clerk saves \$6. What per cent of the salary does he save?

2. In a school of 520 pupils 45% are girls. How many girls are in the school?

3. The following pieces of furniture were sold at a discount of 15%: a couch \$48.50; a dining room set \$250; 6 chairs \$24.50 each; a dresser \$68.50. What was the selling price of each?

4. Furs are advertised for the August sale at a discount of 20%. What is the price of a fur coat marked \$285?

5. A commission merchant sells \$265.50 of hay for a farmer, charging a commission of 6%. How much money does the farmer receive?

6. A grocer buys 144 packages of breakfast food for \$12.96. How much must he charge per package to make at least 25% profit?

7. A shoe manufacturer sells his goods at a profit of 15%. How much is his profit on 2300 pairs of shoes if it cost \$3.63 a pair to make and sell them?

8. The principal of a junior high school, in an assembly talk, said that since last year the number of pupils had increased $3\frac{1}{3}$%. If there were 648 pupils in school last year, how many are there this year?

9. A boy clerks for \$12 a week and a 3% commission on the goods he sells. During one week he sold \$425 worth of goods. What was his income?

176. Business problems.

1. A wholesale grocer bought 450 bags of coffee, each of which weighed 130 pounds, at 28 cents per pound. What was the cost of the whole order?

2. A salesman sold goods valued at $15,450 in one year. If his commission was $12\frac{1}{2}\%$ of the sales, how much did he earn?

3. Cloth damaged by water was sold at $4.10 a yard. Express the reduction in per cent, if the regular price had been $5.60.

4. Two men form a partnership, putting $3800 and $6000 into the business respectively. At the end of the year they made a profit of $3528. How should this be divided between them?

5. A wholesale house quotes the price on silk $2.25 a yard less discounts 25% and 5%. Another quotes the same silk $2.25 a yard less discounts 20% and 10% and an additional 1% for cash. Which is offering the better bargain?

6. An agent drove through the country and bought 7500 bushels of potatoes at $.90 a bushel. He received a commission of $2\frac{1}{2}\%$ for doing the buying, and $34.25 for expenses. How much money did he earn?

7. A boy supplied his neighbors with fresh eggs which were sent to him from his uncle's farm. He sold a 30-dozen crate a week with a profit of 8 cents a dozen, out of which he had to pay 70 cents express charges. What was his profit for the year (52 weeks)?

8. Two shoe stores advertise a sale on the same shoe which ordinarily sells at $5.50. One advertises a discount of 15%, the other a reduction to $4.50. Which is the better offer?

177. Rate of traveling.

1. An automobile traveled a distance of 144 miles in $4\frac{3}{4}$ hours. At what rate did it travel?

2. A train leaves Chicago at 7 A.M. and arrives at St. Louis at 2:45 P.M. If the distance from Chicago to St. Louis is 290 miles, what is the average rate of travel of the train?

3. How long does it take a train to go from San Francisco to New York City, a distance of about 3191 miles, if the train averages 25 miles per hour?

4. One of the big trans-Atlantic steamers traveled 676 knots in 24 hours. What was the average speed if a knot is counted as equal to 1.15 miles?

178. Shop problems.

1. When cooling to normal temperature a steel casting shrinks $\frac{1}{48}$ of the length. What must be the length of a red-hot casting which is to be 38 inches long when cooled?

2. A board is reduced from $2\frac{1}{2}$ inches in thickness to $1\frac{7}{8}$ inches by running it through a planer. How much is taken off on one side, if the board is planed on both sides?

3. If 75% of a casting is copper and the remainder tin, what is the weight of copper in a casting weighing 436 pounds?

4. If the cost of 6 drills is \$7.62, what is the cost of 4 drills?

5. A steel plate is to be made $\frac{3}{4}$ as wide as long. If the length is to be 11.6 inches, what is the required width?

6. If one iron strip is $\frac{1}{8}$ inch thick and another $\frac{5}{32}$ inch thick, which has the greater thickness?

Tests in the Fundamental Operations and Problems

179. When to take these tests. The tests below are to help you test yourself as to speed and accuracy in computation. It is suggested that you take the tests at the beginning of the year and keep a record of the number of problems worked correctly within the time specified in each test. Take the tests again at the end of the semester and at the end of the year. Your records will show the progress made.

Work all exercises in *rows*. In each test place a strip of paper under *one row at a time*, and write on it all work done and the results. The tests contain more exercises than you will be able to do in the time stated.

Test I. Addition: Time, 3 Minutes

7862	6809	8941	5917
5013	7623	7910	4814
1761	5299	9845	9007
5872	6601	8522	6975
3739	3496	1046	1227

6772	7864	8758	2462
6028	7883	2350	9869
6535	8240	3197	4572
2340	9869	2338	6420
2319	6794	5917	6772

1247	4319	6794	3293
3573	2358	5420	7805
1081	5795	4570	7642
7805	4314	8028	7803
9864	1249	8758	2462

Test II. Subtraction: Time, $1\frac{1}{2}$ Minutes

739	1852	975	1087
367	948	906	821
516	962	508	1371
239	325	447	843
1284	730	1853	897
966	508	162	258
1910	735	1056	877
361	478	591	618
1190	619	831	954
739	257	360	483

Test III. Multiplication: Time, 3 Minutes

4857	5718	6942
36	92	58
4065	9625	6123
47	23	64
7486	9027	1253
75	89	38
5376	3786	5492
76	49	53

Test IV. Division: Time, 4 Minutes

$73\overline{)6278}$	$79\overline{)36893}$	$58\overline{)27608}$
$98\overline{)46844}$	$68\overline{)31824}$	$96\overline{)56064}$
$28\overline{)21980}$	$52\overline{)62504}$	$89\overline{)25365}$
$23\overline{)71369}$	$76\overline{)36708}$	$40\overline{)32304}$

Test V. Addition and Subtraction of Fractions: Time, 2 Minutes

(Reduce all answers to simplest form)

$\frac{1}{8}+\frac{1}{4}=$	$\frac{1}{2}+\frac{2}{3}=$	$\frac{3}{4}-\frac{1}{2}=$
$\frac{3}{4}-\frac{1}{3}=$	$\frac{5}{6}+\frac{2}{3}=$	$\frac{2}{3}-\frac{1}{4}=$
$\frac{3}{4}-\frac{2}{7}=$	$\frac{3}{5}+\frac{1}{2}=$	$\frac{3}{4}+\frac{1}{2}=$
$\frac{3}{5}-\frac{1}{2}=$	$\frac{2}{3}-\frac{1}{5}=$	$\frac{1}{6}+\frac{1}{5}=$
$\frac{3}{4}-\frac{2}{5}=$	$\frac{1}{3}+\frac{4}{9}=$	$\frac{2}{5}+\frac{2}{7}=$
$\frac{2}{3}-\frac{3}{5}=$	$\frac{3}{8}-\frac{1}{5}=$	$\frac{5}{12}+\frac{2}{3}=$

Test VI. Multiplication and Division of Fractions: Time, 2 Minutes

(Reduce all answers to simplest form)

$\frac{4}{7}\times\frac{2}{3}=$	$\frac{7}{12}\times\frac{3}{7}=$	$\frac{4}{7}\div\frac{2}{3}=$
$\frac{1}{6}\div\frac{3}{8}=$	$\frac{7}{10}\div\frac{4}{5}=$	$\frac{1}{3}\times\frac{3}{8}=$
$\frac{2}{5}\times\frac{3}{4}=$	$\frac{2}{3}\div\frac{8}{9}=$	$\frac{5}{11}\div\frac{5}{6}=$
$\frac{7}{12}\times\frac{4}{9}=$	$\frac{1}{4}\times\frac{1}{5}=$	$\frac{1}{3}\div\frac{1}{6}=$
$\frac{5}{12}\times\frac{3}{5}=$	$\frac{2}{5}\div\frac{3}{7}=$	$\frac{1}{4}\div\frac{3}{8}=$
$\frac{2}{3}\times\frac{3}{4}=$	$\frac{5}{7}\times\frac{14}{15}=$	$\frac{3}{8}\div\frac{7}{12}=$

Test VII. Placing the Decimal Point in Multiplication: Time, 1 Minute

1.75	×36.9	=	64575	38.608	×5.406	=	208714848
.12	×.12	=	0144	39.2	×9.03	=	353976
16.5	×2.85	=	47025	305.7	×0.76	=	232332
42.08	×5.62	=	2264896	3.45	×16.3	=	56235
42.1	×64.9	=	273229	.46	×.002	=	92
8.5	×5468.	=	464780	19.3	×40.6	=	78358
58.2	×10.8	=	62856	43.5	×9.03	=	392805
.8	×.03	=	24	5.06	×84.7	=	428582
40.3	×5.65	=	227695	94.80	×7.08	=	6711840
47.58	×2.5	=	118950	12.08	×.365	=	440920
57.3	×30.3	=	173619	712.3	× .42	=	299166

Test VIII. Placing the Decimal Point in Division: Time, $1\frac{1}{2}$ Minutes

$.03\overline{)16.2} = 54$	$.04\overline{).348} = 87$
$.06\overline{)7.44} = 124$	$.03\overline{)89.1} = 297$
$.02\overline{).144} = 72$	$.01\overline{)5.48} = 548$
$.03\overline{)47.4} = 158$	$.07\overline{).238} = 34$
$.09\overline{)5.76} = 64$	$.05\overline{).415} = 83$
$.02\overline{).748} = 374$	$.04\overline{)87.6} = 219$
$.09\overline{)94.5} = 105$	$.09\overline{)3.42} = 38$
$.04\overline{)9.84} = 246$	$.05\overline{).965} = 193$
$.07\overline{)1.82} = 26$	$.06\overline{)51.0} = 85$
$.08\overline{).952} = 119$	$.05\overline{)6.85} = 137$
$.08\overline{)40.8} = 51$	$.06\overline{).288} = 48$
$.07\overline{)8.61} = 123$	$.08\overline{)44.8} = 56$

Test IX. Arithmetical Problems: Time, 6 Minutes

(Problems without Numbers)

Directions: The following is a *sample* problem without numbers. Read it carefully and then think how you would solve it:

If you know the number of crates of oranges bought by a fruit dealer and the price paid for them, how would you find the cost of a single crate?

Answer: Divide price paid by number of crates.

On the following pages are *twelve* problems without numbers, like the sample. Try them in order, but do not spend too much time on any one problem. In telling how you would solve a problem without numbers, first write the process, as *divide;* and then the known facts in their proper relation, as *price paid by number of crates.* Note again the answer to the sample problem, *divide price paid by number of crates.*

1. A submarine made a voyage of a given number of miles, going a certain number of miles under water and the remainder of the distance on the surface. How would you find the distance traveled on the surface?

(Answer)

2. A grocer received a bill giving the number of pounds shipped and the cost of an order of sugar. If you were given this bill, how would you find the cost of the sugar per pound?

(Answer)

3. If you know the weight of dough required to make a single loaf of bread, how would you find the weight of the dough a baker must prepare to make a given number of similar loaves?

(Answer)

4. A fruit dealer bought a stalk of bananas containing a certain number of dozens for a certain price. How would you find the cost per dozen?

(Answer)

5. A grocer sells a certain number of pounds of coffee per day. If you know the profit he makes per pound, how would you find his daily profits on coffee?

(Answer)

6. If you know the number of bricks carried by a hod carrier at a load and the weight of the load in pounds, how would you find the weight of a single brick?

(Answer)

7. A man bought a house and lot for a certain price. If he paid a certain amount in cash and gave a mortgage for the rest, how would you find the amount of the mortgage?

(Answer)

8. If you know the number of hours required for a train to travel from one city to another and the number of miles it makes an hour, how would you find the distance between the cities?

(Answer)

9. If you know the number of bushels of corn grown by a farmer on a field of a given number of acres, how would you find the average yield per acre?

(Answer)

10. A boy earned a given sum last week selling newspapers. If he gave a certain part of his earnings to his mother and deposited the remainder in the bank, how would you find the amount he deposited in the bank?

(Answer)

11. The state expects to build a certain number of miles of hard roads this year. If you know the average cost per mile of such road, how would you find the amount of money the state expects to spend for hard roads this year?

(Answer)

12. A man bought a chicken for Sunday dinner. If you know the weight of the chicken and the price paid per pound, how would you find the amount paid for the chicken?

(Answer)

Test X. Arithmetical Problems: Time, 4 Minutes

(Problems with Numbers)

1. My house is 26 feet high, and my flagstaff is four times as high. How high is my flagstaff?

2. I have $629 in the bank. How much more will I have to deposit in order to make my account $1000?

3. A man who earns $1296 per year, makes how much per month?

4. John is 13 years old. Twenty-five years ago, his father was the same age that John is now. How old is the father now?

5. My milk bill was $3.20 last month. I pay 8 cents a pint. How many gallons of milk did I use last month?

6. Massachusetts was settled in 1620. How old was it in 1776?

7. A man sold one horse for $145 and another for $182. How much did he get for both?

8. Potatoes are worth 45 cents a peck. How much will I pay for 8 pecks?

9. A school building has 12 classrooms. The number of pupils in each room averages 36. How many pupils are in the school?

10. A wagonload of coal weighs 4700 pounds. The wagon weighs 1200 pounds. What does the coal weigh?

11. A grocer sells butter at 52 cents, tea at 60 cents, and coffee at 35 cents per pound. How much will I pay if I buy one pound of each?

12. A crate of oranges cost $6.80. If each dozen is worth 40 cents how many dozen of oranges are in the crate?

13. With 6 working days to a week, how much will a man earn in one week at $4.50 per day?

14. There are 328 books in one case in a classroom, and 197 books in the other case. How many books are in both?

15. A boy in the kindergarten found a bag containing 62 marbles. He gave his brother 15. How many did he keep?

16. Nine boys furnished a clubroom, each paying the same amount. Find how much each paid, if the total cost was $11.25.

17. From a flock of 320 sheep, 176 were sold. How many remained?

18. I have $525 in the bank, $125 in my safe, and $3.50 in my pocket. How much money have I altogether?

19. How much will 8 yards of picture wire cost at 3 cents per foot?

20. At Christmas time a factory owner gave away 288 pounds of candy. If each employee received 4 pounds, how many were employed in the factory?

TABLES FOR REFERENCE

Measure for Length

English System

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	= 1 rod (rd.)
320 rods	= 1 mile (mi.)

1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63,360 in.

Metric System

10 millimeters (mm.)	= 1 centimeter (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 meter (m.)
1 meter	= 39.37 inches
1 yard	= .9144 meter

Surveyor's Units of Length

7.92 in.	= 1 link
25 links	= 1 rod
100 links = 4 rd.	= 1 chain
80 chains = 5280 ft.	= 1 mile

Other Units of Length

4 in.	= 1 hand (used in measuring height of horses)
6 ft.	= 1 fathom (used in measuring depth of water)
1.15 mi.	= 1 knot (used in measuring distances at sea)

Counting

12 units	= 1 dozen (doz.)
12 dozen	= 1 gross (gro.)
24 sheets of paper	= 1 quire
500 sheets	= 1 ream

United States Money

10 mills (m.)	= 1 cent (ct. or ¢)
10 cents	= 1 dime (di.)
10 dimes	= 1 dollar ($)
100 cents	= 1 dollar

Measure for Angles and Arcs

60 seconds ($''$)	= 1 minute ($'$)
60 minutes	= 1 degree (°)
$69\frac{1}{6}$ miles	= 1 degree of latitude

Measure for Time

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 days	= 1 common year (yr.)
366 days	= 1 leap year
12 months	= 1 year
360 days	= 1 commercial year
10 years	= 1 decade
100 years	= 1 century

Weights per Bushel

Corn on ear	=70 lb.
Potatoes, wheat	=60 lb.
Onions	=57 lb.
Rye, shelled corn	=56 lb.
Turnips, sweet potatoes	=55 lb.
Apples, peaches	=48 lb.
Barley, buckwheat	=48 lb.
Oats	=32 lb.

Dry Measures

2 pints	=1 quart (qt.)
8 quarts	=1 peck (pk.)
4 pecks	=1 bushel (bu.)

Weights

16 ounces	=1 pound (lb.)
100 pounds	=1 hundredweight (cwt.)
2000 pounds	=1 ton (t.)

Formulas

Angles

1. Sum of the angles of a triangle: $a+b+c=180$
2. Complementary angles: $a+b = 90$
3. Supplementary angles: $a+b = 180$

Circle

Circumference $=2\pi r=\pi d$ $\pi=3.14159$

Percentage

1. Interest, $i=\dfrac{prt}{100}$
2. Percentage, $p=\dfrac{rb}{100}$

INDEX

(*References are to pages.*)